Aviation in a sustainable world

Final Report: Prioritisation of Airframe and Engine Technologies

Part 2 Technology Prioritisation

Main thematic area: Aircraft Systems

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About Omega

Omega is a one-stop-shop providing impartial world-class academic expertise on the environmental issues facing aviation to the wider aviation sector, Government, NGO's and society as a whole. Its aim is independent knowledge transfer work and innovative solutions for a greener aviation future. Omega's areas of expertise include climate change, local air quality, noise, aircraft systems, aircraft operations, alternative fuels, demand and mitigation policies.

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Objective

This study examines the impact of advances, improvements and changes in both engine and airframe technology and assesses them in terms of their potential to mitigate the environmental impact of aviation. It will identify which technologies offer the best improvements when all the relevant issues are considered.

Background

The list of candidate technologies that could reduce aviation's environmental impact is large. However, it is difficult to quantify the net realisable benefit of an improvement in a single characteristic at the system level that includes the airframe, the engine and the atmosphere. The challenge is to identify those technologies that offer the largest, or the quickest or the most cost effective benefits in terms of a range of specified environmental impacts. This study uses a simple, accurate, analytic system model capable of estimating the overall impact of a given improvement in technology. The output is tailored to provide support for the planning of future research and development programmes for aerospace technologies. Examples of the areas that will be assessed include - the implementation of carbon fibre structure, improvements to high lift systems, improvements in transonic aerofoil design, drag reduction technologies, improved propulsive efficiency, improved combustion and engine component technology. The output will be judged in terms of a number of target metrics e.g. fuel burn per passenger seat mile, payload fraction. NOX production and propensity to form contrails. Other optimisation parameters may be generated in the course of the study.

Knowledge transfer

An important impact of this research will be that the development of a set of technological pathways and strategic responses that could form a framework for further informed discussion between industry representatives and policymakers, as to the likely impact of different policy options. The uptake of different technological options for reducing the environmental impacts of aviation depends significantly on the extent to which these make commercial and strategic sense for actors within the aviation industry, including manufacturers, airlines and airport operators. It is vital to develop a realistic understanding of the possible range of responses by firms in the aviation industry to the incentives that policies will create. These responses would include decisions on investment in new technologies for aircraft.

The purpose of this report is to describe the potential role of engine and airframe technology in the mitigation of aviation's impact on the environment and to describe mathematical models that permit the quantification of technological improvements at the system level. It is aimed at two groups of readers. The first is the high level, essentially non-technical, reader who needs a general description of the issues and their context. This is covered in the first half of the report. The second is the technical specialist who needs to assess the impact of an improvement in the performance of a component, or a sub-system, on total system parameters such as fuel burn per passenger seat mile.

1 Introduction

The purpose of air transport is to take passengers and cargo from one place to another for commercial gain. This objective is achieved by the air transport "system". The "system" consists of a number of airline operators dealing directly with the customers and operating fleets of aircraft on a range of routes. These operators use the facilities provided at a number of airports and a range of national air traffic services in order to get their aircraft from their departure points to their destinations. In addition, to ensure a high standard of safety, the "system" is tightly regulated and the whole activity is subject to international agreements and international law. Therefore, it is something of an understatement to say that the international aviation business is exceedingly complex. However, it is important to recognise from the outset that the "system" has many important and sometimes subtle interlinkages. This means that, in order to asses the net effect of a change in any part, the response of the whole system must be considered.

From the point of view of environmental impact, the common perception is that this is governed by only the electro-mechanical elements of the "system" i.e. the aircraft itself and, particularly, the engine. However, whilst it is true to say that the engine is responsible for all the gaseous emissions, most of the particulate emissions and most of the noise, the quantity, the location and the impact, on both the environment and on human health, is influenced to a very large extent by other components of the total "system".

The primary purpose of this work is to asses the potential effect of advances in aircraft technology on aviation's impact upon climate. Therefore, the focus is on the high altitude emissions of the various products of combustion. However, it should be noted that the environmental benefits of improved technology are not solely related to the aircraft and its engines. There is clearly something to be gained by applying new technology to other parts of the "system" e.g. to air traffic management.

In work that has already been published (reference 1), it has been argued that the key parameter linking the economic performance and the environmental impact of an aircraft is the ratio of the energy liberated by burning the fuel required for the flight to the work done by carrying the cargo (human and inert) over the distance travelled. This is the energy to revenue work ratio (ETRW) i.e.

$$ETRW = \frac{MMF.LCV}{MP.g.R}$$

where

MP = payload mass (kg)

g = acceleration due to gravity (m/sec/sec)

R = the great circle distance between departure point and destination (m)

MMF = the mass of fuel consumed (kg)

LCV = the fuel lower calorific value (J/kg)

Clearly, for economic reasons, the smaller the value of this ratio the lower the cost of energy for a given mission and the lower the aircraft's direct operating cost (DOC), since

$$\frac{\textit{revenue generated}}{\textit{cost of energy used}} = \frac{A}{B}\left(\frac{1}{ETRW}\right)$$

where

A = revenue/unit payload weight/unit distance travelled

And

B = fuel cost/unit of energy released

Equally, the smaller the value of this ratio, the smaller the total quantity of emissions produced during a given mission and, hence, the lower the environmental impact, i.e.

$$\frac{emissions\ mass.LCV}{useful\ work\ done} = \left(ETRW\right)\left(a.EI_{CO2} + b.EI_{NOX} + c.EI_{H2O} + d.EI_{SOX} + ...\right)$$

where the constants *a, b, c, d,* etc. and the Emission Indices depend upon both the fuel being used and the level of engine technology. For example, if the fuel is oil-derived kerosene, *a, b, c* and *d* will be unity, but, if biomass-derived kerosene is being used, *a* could be less than unity, since the net carbon dioxide addition to the atmosphere could be lower than that for oil-derived kerosene. Similarly, if the fuel is hydrogen then the Emission Indices for carbon dioxide and SOX would be zero.

However, there is another phenomenon that contributes to climate change via a direct impact upon global warming and that is the formation of contrails. Contrails can only be formed when an aircraft passes through air that is supersaturated with respect to ice. In the northern latitudes, where most of the air transport routes are, air that is supersaturated with respect to ice occurs at altitudes close to the tropopause i.e. somewhere in the range 30000 to 40000 feet. Present generation aircraft tend to have their best fuel burn altitudes within this range and, consequently, they produce a great many contrails. Therefore, if the number of contrails is to be reduced, future aircraft must cruise at altitudes below 30,000 feet.

In the past, it has been common practise to design aircraft for minimum cost of ownership or minimum direct operating cost. However, the cost of ownership and *DOC* calculations include many elements that are linked to the costs of money, services, fuel and labour. When an aircraft is being designed the values of all these elements are known. However, what is not known is how the cost of these components will vary with time and, since a modern aircraft has a service life of about 25 years, it is possible that enormous economic variations will be experienced and these will alter the operating economics of a given aircraft. Even if estimates of future costs are made, these are notoriously unreliable. Therefore, whilst we can be confident that the "Laws of Physics" will not change over the working life of an aircraft the same cannot be said about the "laws of economics". Since *ETRW* is dependent solely upon the Laws of Physics and since it has direct relevance to environmental impact, it is truly future proof and, as such, it is a better design parameter than cost of ownership and *DOC*.

In the context of the public debate surrounding aviation, there is a need for accurate information and a need for an appreciation of the issues and the arguments that lie behind them. However, there are a number of reasons why such information and understanding can be slow to emerge. Firstly, the information may not lie in one place or even with one stakeholder. Secondly, aviation is a highly technically complex business. The information that is needed may not be available in a simple form and the people who may be tasked with producing it may not know how to generate it. Thirdly, and perhaps most importantly, much of the information, many of the issues and some of the arguments may be commercially sensitive and stakeholders may find it difficult, or impossible, to discuss critically important matters in a public forum. Worse still, a reluctance to engage in public discussion can easily be interpreted as evasive or suspicious behaviour. Either way, the absence of important information or a clearly expressed argument, or a loss of credibility in a key stakeholder could lead to poor decisions and bad policy making.

The Omega approach is to use academia as an independent source of information and knowledge. By developing analytically rigorous tools that do not use any proprietary, or commercially sensitive, data and are subject to peer review and published in the open literature, the academic network can supply both the information and the arguments. This would be a benefit to all stakeholders and would allow the aviation sector to fight its corner in a way that is currently very difficult to do. The risk is that some of the messages coming from the academic domain may not support the industry's view or its wishes. Nevertheless, in this event, the industry will have the opportunity to challenge and question.

1.1 Factors affecting *ETRW*

It has been shown in reference 1 that a given aircraft exhibits its best *ETRW* when it is carrying the maximum possible cargo mass for the maximum possible distance i.e. it is taking off at the maximum zero fuel mass and the maximum permitted take-off mass. However, the actual value of *ETRW* at this condition is determined by the design process.

In the conventional business model, an aircraft operator seeks to satisfy a market demand for transport capacity between two places. The operator's needs will define a number of "market requirements" for the aircraft. These will be (reference 2) –

- Design payload
- Design range
- Cargo capacity
- Operating economics
- Engine options and timing

Since cargo is usually, but not always, a secondary consideration when a new aeroplane is designed, it is the number of passengers to be carried that determines the design payload. A specification of the number of passengers to be accommodated effectively fixes the size of the fuselage. This is because of the anthropological requirement to provide about 0.85 m^2 of fuselage floor space for each passenger and, with the fuselage floor area determined, the size and the mass of the fuselage are effectively fixed.

Behind the fixed market requirements are the "first order airline requirements". These are likely to be

- Sea level field performance (takeoff and landing distances)
- High altitude field performance (does the aircraft need to use airports like Denver (5400m), Johannesburg (5550m) or Mexico City (7340m)?)
- Wing span limitation
- Noise
- Design cruise Mach number

The distance between the two places determines the design range and the facilities available at the airports define the required field performance, the bay size and the noise restrictions. However, the cruise Mach number may depend upon a number of factors. If the distance between the two places is very large, passenger comfort may dictate a high cruise speed. On shorter trips, the cruise speed may be determined by operational economic issues, timetabling (how many trips per day) or fuel economy.

Clearly, the requirements of an airline are commercially sensitive. Moreover, the design "know how" and the technology that the airframe and engine manufacturers use to satisfy their customers needs is the source of their competitive advantage. Therefore, it is very difficult to obtain detailed information on these topics and, even if information was obtained, it may not be useable in open discussions or in academic publications.

It is at this point that the Omega tools are needed. In this study, we have developed two initial models, one for the engine and one for the aircraft. These models are simple, analytic, comprehensive and interlinked. Their development has been overseen by established experts in the field of aircraft and engine design and where possible, they have been compared with publicly available data and, in some cases, comparisons with the results of more sophisticated models have been possible. These models are described in outline in the technical annex.

These models have been used in other Omega studies, most notably the JETCLIM work where the output has been used to understand why aircraft currently fly in the parts of the atmosphere where the air may be supersaturated with respect to ice. Contributions have also been made to the mitigation studies and the airline economic studies. In the present context, the models have been used to replicate the most fundament aspects of aircraft design.

The design process for a civil aircraft is a complex process. However by using the models it is possible to reproduce some of the key elements and to draw conclusions. With the size of the fuselage fixed, the next consideration is the size of the wing and the engines. The aerodynamic efficiency of the wing depends upon its shape rather than its size. If we consider an aircraft with a wing with a specified planform, i.e. fixed values of the aspect ratio (span^2/area), thickness to chord ratio, taper ratio (tip chord/root chord) and sweep-back angle, cruising at a specified Mach number and altitude, there is a particular value of the wing area (wing size) at which the fuel burn for the trip is a minimum. In addition, in order to cruise at a specified altitude the aircraft must have engines with an appropriate amount of power. As the cruise altitude increases, more powerful and, hence, heavier engines are required. This needs to be taken into account as the cruise altitude and the cruise Mach number change. However, to a first approximation the thrust requirement is independent of the number of engines used.

For a given Mach number, the size of wing required for minimum fuel burn varies with cruise altitude and there is a particular cruise altitude, and a particular wing size, for which the fuel burn for the trip in an absolute minimum. The size of this "best" wing varies with the number of passengers being carried. However, the ratio of the maximum payload mass to wing area, MMP/S_{ref}, at this condition is approximately 250 kg/m^2 and this ratio is only weakly dependent upon the number of passengers being carried. For cruise Mach numbers close to 0.8, the corresponding "best" cruise altitude is in the range 20,000 to 25,000 feet. This condition defines the aircraft with the minimum *ETRW*.

Unfortunately, cruise is not the only phase of the flight that needs to be considered in the design process. Clearly, for an aircraft to be able to travel between two points, those points must have airports and the aircraft must be able to take off and land on the runways that are available. Therefore, the length of tarmac provided sets an important and unavoidable constraint for a practical aircraft.

When an aircraft is approaching a runway to land, the primary consideration is the speed that it has when is reaches a point 50 feet altitude above start of the runway with the engines at idle. This is sometimes called the "threshold" speed. The choice of threshold speed is essentially arbitrary. However, on safety grounds, the threshold speed should not exceed about 75 m/sec (145 kts). In addition, the higher the threshold speed, the longer the braking distance and the greater the length of tarmac required to stop the aircraft. With a standard three degree glide slope and current brake technology, an aircraft with this threshold speed will come to a stop on a wet runway in less that 1800m. In terms of access, an analysis of the available runways shows that, if an aircraft can stop within a distance of 1800 m, it will be able to land at 90% of the world's airports.

Since the airworthiness regulations (JAR 25) require that the threshold speed must be greater that 1.3 times the stalling speed in the landing configuration, i.e. with full landing flap set. For a given level of flap technology, this requirement and the need to limit the threshold speed combine to set a minimum permissible value for the wing area. Therefore, if the maximum permitted landing mass of the aircraft is MLM, then

$$\frac{MLM}{S_{ref}} \leq 200\left(Cl_{land}\right)_{max}.$$

Since a typical maximum value for the lift coefficient with full flap is 2.75, this means that in order to land on a runway 1800 m long, the wing "loading", MLM/S_{ref}, must be less than 550 kg/m^2. In addition, since maximum landing mass is typically between 3 to 4 times the maximum payload mass,

$$\frac{MMP}{S_{ref}} \leq 150 \text{ kg/m}^2$$

Therefore, even with the provision of a powerful high lift system, in order to meet the landing constraints of a safe threshold speed and a safe margin of speed above the stall, the aircraft must have a wing that has a much bigger surface area than that necessary for best fuel burn. Consequently, the aircraft is heavier, the engines are more powerful and the cruise altitude for best fuel burn increases by about 10,000 feet. Therefore, the landing constraint imposes a significant fuel penalty. This is of the order 10-20% and depends upon the number of passengers carried. In addition, the need to increase cruise altitude places the aircraft in regions of the atmosphere where it is most likely to encounter air that is supersaturated with respect to ice i.e. where it will produce contrails.

In addition to being able to land within a specified distance at the maximum landing mass, the aircraft also has to be able to take off again. The take-off field performance is a much more complex process than the landing field performance and, under certain conditions, the required take-off performance can determine both the wing area and the engine power. Take off involves a ground run, a rotation, a lift off and a climb to a height of 35 feet; with the total horizontal distance covered whilst performing these four elements being designated the "take-off distance". However, if the aircraft is multi-engined and it suffers a total failure of one engine during the ground run, depending upon the speed at which the engine fails, the pilot will elect to do one of two things. At low forward speeds, the pilot will still have sufficient tarmac ahead to be able to use the brakes to stop the aircraft within the boundaries of the runway. However, at higher speeds, the pilot may be able to continue the take-off with the remaining engines at full power. For any given aircraft, there is a particular runway length for which the total horizontal distance covered is the same for both options. This is the so-called "balanced" field length and it is the distance usually used to determine whether an aircraft can operate from a given length of runway i.e. it is the aircraft's required take-off distance. The balanced field length depends upon the mass of the aircraft and it has its largest value when the aircraft is at its maximum permitted take-off mass, *MMTO*.

As already noted, the calculation of the balanced field length is complicated. However, the distance depends upon the take-off mass (*MTO*), the area of the wing, S_{ref}, the total engine thrust, *Foo,* the number of engines and the flap setting. The greater the amount of flap deflection the shorter the take-off run will be and, since the aircraft needs a powerful flap system for landing, it seems logical to extract the maximum benefit from the flap system for take off. However, the amount of flap that can be used during take off is determined by the airworthiness regulations that are applicable to the "second stage climb" (SSC). This phase begins as the aircraft exceeds an altitude of 35 feet, with the engines at take-off power, the flaps at the take-off setting and the undercarriage retracted and ends when the aircraft passes an altitude of 400 feet. During the second stage climb, the airworthiness regulations require that, in the event of a multi-engined aircraft suffering a complete loss of power in one engine, the aircraft must be able to maintain a minimum specified climb gradient whilst flying faster than 1.2 times the stalling speed. These gradients are 2.4% for a twin engined aircraft, 2.7% for a three engined aircraft and 3.0% for a four engined aircraft. Since the take-off distance is dominated by the ground run, the distance will be shortest when the take off speed is as low as possible and the lowest take off speed occurs when the maximum usable lift coefficient is as high as possible. However, the maximum useable lift coefficient is determined by the airworthiness requirements for the second stage climb with one engine failed.

The maximum useable lift coefficient is the one that just delivers the minimum permissible SSC climb gradient in the event of an engine failure. The greater the thrust available, the greater the maximum usable lift coefficient and the shorter the take off distance. Moreover, since the SSC speed must be at least 20% larger than the stall speed, the maximum usable lift coefficient must be less than (1/1.44) times the maximum available lift coefficient (stall condition). Therefore, since the maximum available lift coefficient is a function of the flap setting, the greater the installed engine thrust the more flap can be used for take off, with the shortest take off distance corresponding to maximum flap deflection.

It follows that the installed engine thrust is a very important parameter for the take off run since, the higher the installed thrust, the shorter the ground run to a specified take off speed and, the higher the installed thrust, the lower the minimum permitted take off speed. In addition, it can also be seen that, for a given level of installed thrust, an aircraft with three, or four, engines will have a shorter take off run than an aircraft with two engines. This is because, in the case when an engine has failed, the twin engine aircraft has the lowest thrust. Therefore, in order to generate the required SSC gradient, it must be flying faster i.e. it must take off with a lower flap setting than the three or four engine aircraft.

In operational terms, whilst it is highly desirable to be able to always carry the maximum amount of payload, it is not always necessary to take off at the maximum take-off mass. Therefore, whilst typical landing distances at maximum landing mass are in the range 1400m to 2000m, typical take off distances at maximum take off mass are in the range 2000m to 3000m. Since 65% of the world's airports have runway lengths that are 2500m, or less, this means that an aircraft can take off at maximum take off mass at far fewer airports than it can land on at maximum landing mass.

Over the past 40 years, the twin engine layout has emerged as the dominant configuration for all but the longest range aircraft. The primary reason for this has been economics i.e. it is much cheaper to own and to maintain an aircraft that has two engines than one that has three or four. However, it is now clear that there is an additional environmental penalty for having just two engines. This is because of the problem of achieving a take off distance in the range 2000m to 3000m. For a twin engine configuration this can only be achieved by increasing the wing area over and above the minimum value imposed by the landing speed requirement or by increasing the level of the installed thrust over and above the minimum required by the cruise requirement. Using a larger wing area would increase an already increased *ETRW* and increasing the installed thrust would increase the *ETRW* because the total mass of the aircraft would be increased.

Through these arguments, it is possible to identify the technologies that can have an impact upon *ETRW*. An obvious target is the reduction of the wing area. If, as argued above, the wing area is determined by the requirement to keep the landing speed at a safe level, the wing area requirement can be relaxed and cruise fuel burn reduced by using a flap system that develops a higher maximum lift coefficient that the mechanical flap systems in service today. If the maximum lift coefficient could be increased from today's values of around three to a value of four, the wing area could be reduced by 25%. This would produce an improvement in *ETRW* in the region of 5% and the optimum cruise altitude would be lowered so that the propensity to make contrails would be less. Greater improvements in the maximum lift coefficient would deliver correspondingly larger improvements in *ETRW*. However, reducing the wing area also has the effect of increasing the take off distance. If the aircraft has two engines, the take off distance will have to be corrected by increasing the installed thrust and this will offset the cruise fuel burn improvement somewhat and may increase the noise level during take off. On the other hand, if the aircraft has three or four engines, it may be possible to use the additional maximum lift coefficient capacity of the flap system to reduce the take off distance further. In military aviation, aircraft that are intended to operate from very short runways tend to have four engines and, in some cases, use "blown" flaps to generate very high lift e.g. the Boeing C-17. These existing capabilities could potentially deliver significant *ETRW* and noise benefit with very low technical and commercial risk and they could be introduced quickly.

As identified in reference 1, the *ETRW* is sensitive to the cruise Mach number and the analysis suggests that, if cruise Mach numbers could be reduced to values around 0.7, reductions in *ETRW* in excess of 10% could be achieved without the need to introduce high risk technologies. This would be in addition to the 10%-15% improvement that will come though gradually with the move towards all composite airframes. In addition, the adoption of slightly lower cruising speeds would mean less powerful engines and this may allow noise reductions. A lower cruise Mach number requirement would facilitate the introduction of new propulsion concepts such as the open rotor. Since the potential benefits of this propulsion system are substantial, its early introduction into service would be advantageous.

Therefore, whilst the trade press and the media concentrate their attention on the long term, very high risk technological "solutions" such as laminar flow control or the blended wing body configuration. In practical terms, even under the most favourable economic conditions, these ideas are decades away from entry into service and, consequently, do not offer any benefits in the short or medium term. However, as described in reference 1, it is clear that there is significant potential to reduce aviation's environmental impact by improving the efficiency of airline operations and air traffic management provision. These improvements can be made without any changes to the existing aircraft fleet. In the context of the aircraft themselves, substantial improvements could be achieved in the short to medium term by moving to designs using more than two engines, lowering design cruise speeds to around Mach 0.7 and by improving the performance of those aircraft systems that have a indirect, though potentially powerful, impact on fuel burn. The most important of these is the high lift system. There is potential to improve the performance of existing high lift systems and it is possible to apply existing military high lift technology to civil aircraft. Reducing cruise speeds is a realistic proposition for short haul operations, since most of the fuel that is used for aviation is burned in on flights of less than 500 nm. Reducing cruise speed would also reduce the technical challenges facing the open rotor propulsion concept. This has the potential to reduce fuel burn by 10-15% and reducing the technical challenges could mean lower development costs and an earlier "in service" date. Finally, more benefit could be extracted by optimising designs so that *ETRW* is minimised rather than the traditional *DOC*. All these improvements can be delivered in the short to medium term and do not require the implementation of the exotic or high risk, technologies.

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2 Technical Annex

This technical annex provides a brief description of the models that have been developed as part of the Omega programme

2.1 Airframe

This section describes in outline the mathematical model of a civil aircraft. The model can be used in a number of different modes e.g. for a given mission specification, to find the aircraft and engine combination that delivers the minimum fuel burn or to provide a complete performance model of a particular aircraft when only some of its characteristics are known (reverse engineering). In the present work, the model is used to investigate the relationship between fuel burn and altitude.

A civil aircraft is designed to carry a specific number of passengers, a certain distance at a particular speed and to operate from airport runways of specified size. If, in addition, there is a requirement to burn the smallest amount of fuel whilst carrying out the "mission", then, for a given level of technology, specification of these parameters is sufficient to completely determine the layout the aircraft, its mass and its principal performance characteristics.

In addition to the burning of kerosene and the attendant emissions of carbon dioxide and NOX, aircraft flying at high altitude may, if the atmospheric conditions are appropriate, form a contrail. Contrails contribute directly to the Earth's thermal radiation balance and so this phenomenon needs to be treated as an "emission" when assessing the total environmental impact of aviation.

2.2 Aircraft components

The following analysis is based upon the characteristics of current aircraft. Where empirical information relating to current aircraft is need, this is obtained from published data for the 60 types listed in Appendix 1.

2.2.1 Fuselage

The size of the fuselage is dictated by the number of passengers that it must accommodate. The so called "single class" configuration defines the largest number of passengers that an aircraft can carry. For a given aircraft, the maximum number of passengers permitted in the single class configuration is determined by the airworthiness regulations and is recorded on the aircraft's Type Certificate. Analysis of data from over 60 aircraft currently in service, see figure 1, reveals that in the single class configuration each passenger requires a floor area of about 0.825 m^2. This includes allowance for aisles, galleys, toilets and crew areas.

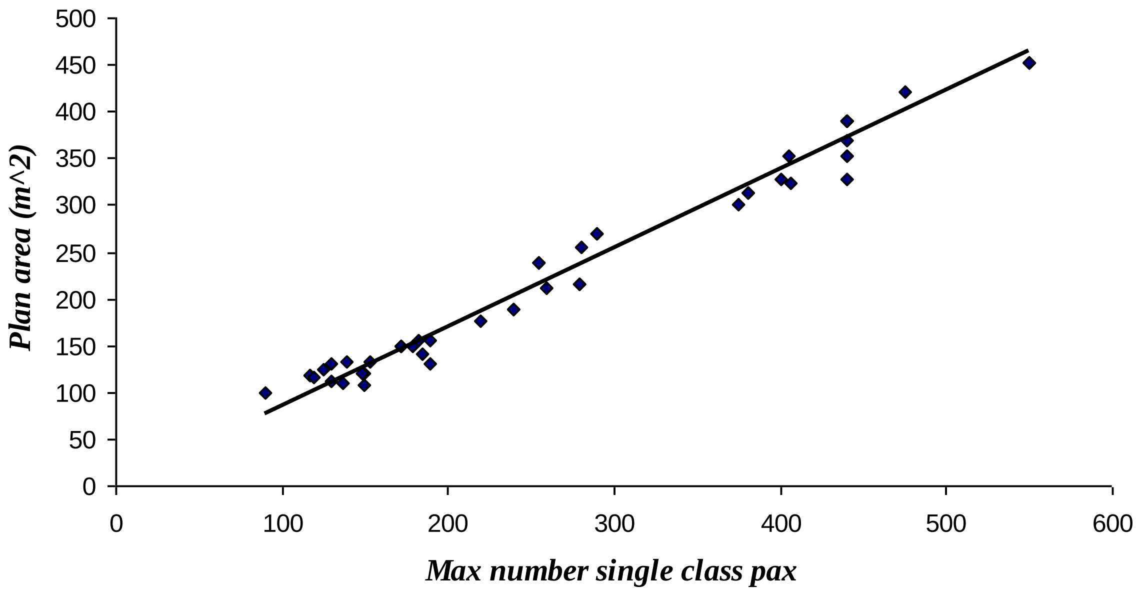


Figure 1 – Variation of fuselage plan area with maximum number of passengers permitted in a single class layout.

Therefore, if the aircraft is to carry *N* passengers in single class, the plan area of the fuselage is given by

$$\frac{S_{plan}}{b^2} = \frac{l_f}{b} \approx \frac{0.825}{b^2} N \,,$$

where l_f is the length of the fuselage and b is its width.

For aircraft with a typical three class layout, the number of passenger N_3 is related to N by

$$N_3 \approx 0.67 N$$

The key geometric characteristic of the fuselage is the slenderness ratio, λ_f. This is defined as -

$$\lambda_f = \frac{2l_f}{(b+h)} = \frac{l_f}{b}\frac{2}{(1+h/b)} = \frac{l_f}{b}\frac{2}{(1+e)} = \frac{2}{(1+e)}\frac{S_{plan}}{b^2}$$

where h is the height of the fuselage and e is the eccentricity of the cross section, (h/b). Once the slenderness ration is known, the volume, Vol_f, and the surface area can be estimated e.g. by using the approximate relations given by Torenbeek (reference 2)

$$\frac{Vol_f}{S_{plan}^{3/2}} \approx \left(\frac{\pi}{4}\right)\left(1-\frac{2}{\lambda_f}\right)\left(\frac{1}{\lambda_f}\frac{2}{(1+e)}\right)^{1/2}$$

and

$$\frac{(S_f)_{wet}}{S_{plan}} \approx \pi\left(1-\frac{2}{\lambda_f}\right)^{2/3}\left(1+\frac{1}{\lambda_f^2}\right) \approx 1.98\lambda_f^{0.14} \,.$$

The payload carried by an aircraft consists of passengers plus luggage and cargo. Analysis of current aircraft reveals that the maximum permissible payload is directly proportional to the maximum number of passengers that can be carried and, on the average, the maximum payload mass per passenger is close to 125 kg. This is shown in figure 2.

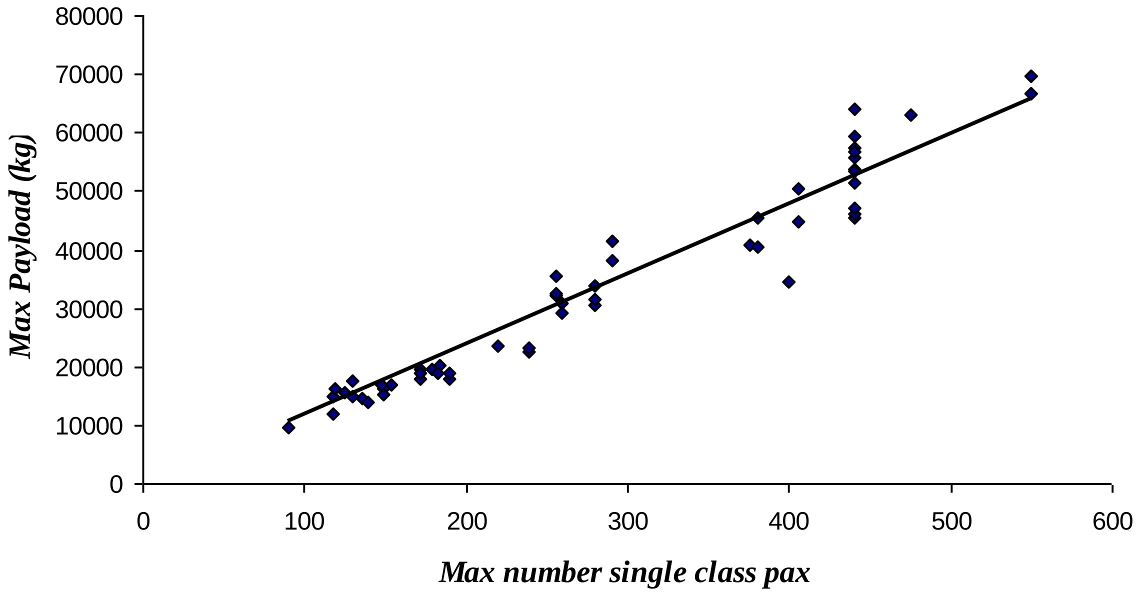


Figure 2 – Variation of maximum permitted payload with maximum permitted number of passengers in a single class configuration.

Therefore, for a market competitive aircraft, the fuselage loading, MMP/S_{plan}, should be about 145 kg/m^2.

According to Howe (reference 3), the mass of a pressurised civil aircraft fuselage may be estimated using the following empirical relation

$$M_{fus} \approx 0.79\, p_{diff} (0.75 + 5.84b)(l_f - 1.5)(b + h)^2 \, (kg)$$

or

$$M_{fus} = 4(0.79)\, p_{diff} (0.75 + 5.84b)\left(1 - \frac{1.5}{\lambda_f}\right)\left(\frac{1+e}{2}\right) S_{plan},$$

where S_{plan} must be given in m^2, the fuselage width, b, must be given in metres and p_{diff} , the cabin maximum working pressure difference, must be given in bars. This expression can be simplified by replacing p_{diff} with $\Delta p/p_0$ where

$$\frac{\Delta p}{p_0} = \frac{p_{cabin} - p_{\infty}}{p_0} = \frac{p_{diff}}{p_0} \,.$$

Here p_0 is the atmospheric pressure at sea level in the International Standard Atmosphere (reference 4)

Hence,

$$\frac{M_{fus}}{MMP} = 0.022\frac{\Delta p}{p_0}(0.75 + 5.84b)\left(1 - \frac{1.5}{\lambda_f}\right)\left(\frac{1+e}{2}\right).$$

Typically, a civil transport aircraft will have a maximum working altitude of about 42,000 feet, whilst the air pressure inside the cabin will be equivalent to atmospheric conditions at about 8,000 feet. Therefore, a typical value for $\Delta p/p_0$ will be 0.59.

In order to minimise the aerodynamic drag on the fuselage, the slenderness ratio, λ_f, is kept within the range 8 to 14, with the lower value being typical of the first generation (unstretched) design. This is achieved by varying the fuselage width, b, according to the number of passengers carried as shown in figure 3,

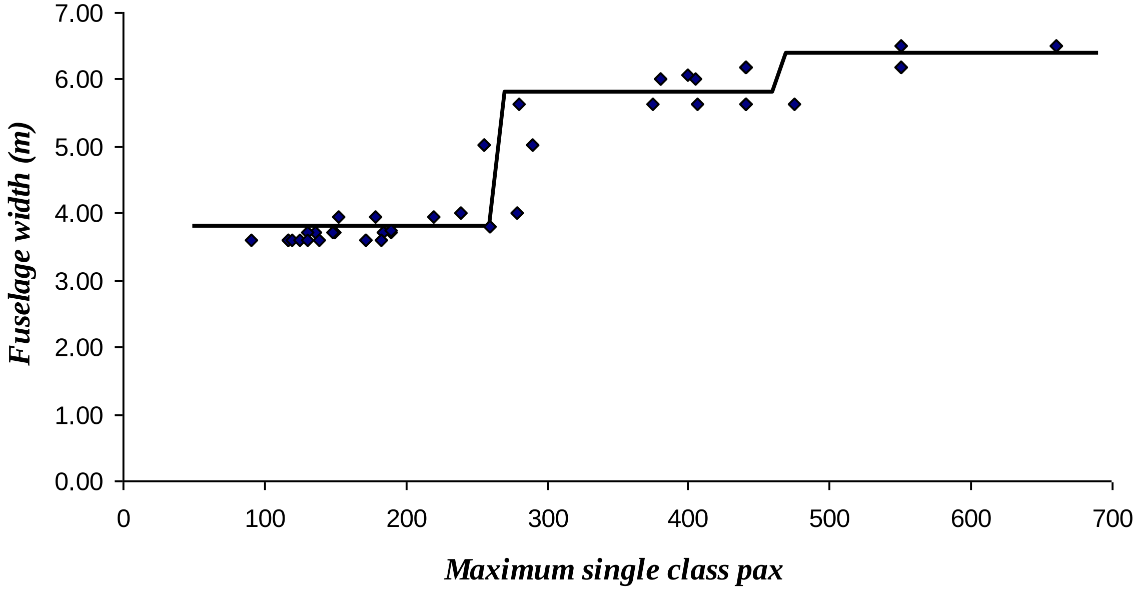


Figure 3 – Variation of fuselage width with the maximum number of passengers permitted in a single class layout.

It follows that, for $N < 265$,

$$b = 3.8\text{ m},\ \frac{S_{plan}}{b^2} = 0.0571N\ ,\ \frac{\left(S_f\right)_{wet}}{S_{plan}} = 1.326\left(\frac{2}{1+e}\right)^{0.14} N^{0.14}\text{ and}$$

$$\frac{M_{fus}}{MMP} = 0.705\frac{\Delta p}{p_0}\left(\frac{1+e}{2}\right)\left(1 - \frac{26.25}{N}\left(\frac{1+e}{2}\right)\right)$$

for, $265 < N < 470$,

$$b = 5.8\text{ m},\ \frac{S_{plan}}{b^2} = 0.0245N\ ,\ \frac{\left(S_f\right)_{wet}}{S_{plan}} = 1.178\left(\frac{2}{1+e}\right)^{0.14} N^{0.14}\text{ and}$$

$$\frac{M_{fus}}{MMP} = 0.963\frac{\Delta p}{p_0}\left(\frac{1+e}{2}\right)\left(1 - \frac{61.16}{N}\left(\frac{1+e}{2}\right)\right)$$

and, for $N > 470$,

$$b = 6.4\text{ m},\ \frac{S_{plan}}{b^2} = 0.0201N\ ,\ \frac{\left(S_f\right)_{wet}}{S_{plan}} = 1.146\left(\frac{2}{1+e}\right)^{0.14} N^{0.14}\text{ and}$$

$$\frac{M_{fus}}{MMP} = 1.041\frac{\Delta p}{p_0}\left(\frac{1+e}{2}\right)\left(1 - \frac{74.47}{N}\left(\frac{1+e}{2}\right)\right).$$

Therefore, once the number of passengers, single class or three class, has been specified the maximum payload, the fuselage geometry and the fuselage structure mass are completely determined.

2.2.2 Lifting Surfaces

The aircraft's lifting surfaces are the wing, the horizontal tail (tailplane) and the vertical tail (fin). The principal function of the wing is to provide aerodynamic lift to support the total weight of the aircraft through all the phases of the flight. A secondary function for the wing is the provision of storage space for the fuel. The tailplane and fin are necessary to give the aircraft stability, to provide balancing moments and to generate additional moments and forces for manoeuvres.

The basic aircraft wing has a nearly trapezoidal planform whose primary geometric characteristics are

- Aspect Ratio, A, - s^2/S_{ref} where s is the wingspan and S_{ref} is the wing gross plan area (including the portion inside the fuselage)
- Taper ratio, λ_w, – the ratio of the streamwise chord at the tip to the streamwise chord at the aircraft centreline
- Thickness to chord ratio, t/C – the ratio of the wing section max thickness to streamwise chord length
- Sweep angle, Λ, – the angle between the ¼ chord line and the normal to the fuselage centreline

These ratios specify the shape of the wing and the size of the wing is determined once S_{ref} is known.

The wing volume, Vol_w, is given by

$$\frac{Vol_w}{S_{ref}^{3/2}} \approx (0.70)\left(\frac{4}{3}\right)\left[\frac{(1+\lambda_w+\lambda_w^2)}{(1+\lambda_w)^2}\right]\left[\frac{(t/c)}{A^{1/2}}\right],$$

where the number "0.70" is a typical value of the aerofoil streamwise cross-sectional area divided by the product of the local maximum thickness and the local chord length. Civil transport aircraft carry their fuel in the wing. Therefore, the wing volume determines the maximum amount of fuel that can be carried. For current aircraft, the maximum available fuel volume is approximately 60% of the total wing volume.

The exposed wing area, S_w, is given by the difference between the gross plan area, S_{ref}, and the area of the wing covered by the fuselage. That portion of the wing area covered by the fuselage is approximately equal to the product of the root chord, C_r, and the passenger cabin width, b. For a trapezoidal wing

$$C_r = \frac{2}{(1+\lambda_w)}\left(\frac{S_{ref}}{A}\right)^{1/2}$$

and

$$b = \left(\frac{S_{plan}}{\lambda_f}\right)^{1/2}.$$

Therefore,

$$\frac{C_r b}{S_{ref}} = \frac{2}{(1+\lambda_w)}\left(\frac{1}{A}\frac{1}{\lambda_f}\right)^{1/2}\left(\frac{S_{plan}}{S_{ref}}\right)^{1/2}.$$

In addition, if S_{plan} and S_{ref} are of similar magnitudes, then

$$\frac{S_{plan}}{S_{ref}} = 1 + \iota$$

and ı is small. Hence,

$$\left(\frac{S_{plan}}{S_{ref}}\right)^{\varpi} \approx 1 + \varpi\iota = 1 + \varpi\left(\frac{S_{plan}}{S_{ref}} - 1\right) = (1-\varpi) + \varpi\left(\frac{S_{plan}}{S_{ref}}\right).$$

It follows that,

$$\frac{C_r b}{S_{ref}} \approx \frac{1}{(1+\lambda_w)}\left(\frac{1}{A}\frac{1}{\lambda_f}\right)^{1/2}\left(1 + \left(\frac{S_{plan}}{S_{ref}}\right)\right)$$

and

$$\frac{S_w}{S_{ref}} \approx \left(1 - \frac{1}{(1+\lambda_w)}\left(\frac{1}{A}\frac{1}{\lambda_f}\right)^{1/2}\right) - \frac{1}{(1+\lambda_w)}\left(\frac{1}{A}\frac{1}{\lambda_f}\right)^{1/2}\left(\frac{S_{plan}}{S_{ref}}\right).$$

Having obtained the general form of the relation, examination of the characteristics of the aircraft listed in the appendix reveals that, on average,

$$\frac{S_w}{S_{ref}} \approx 0.913 - 0.087\left(\frac{S_{plan}}{S_{ref}}\right).$$

The data are given in figure 4

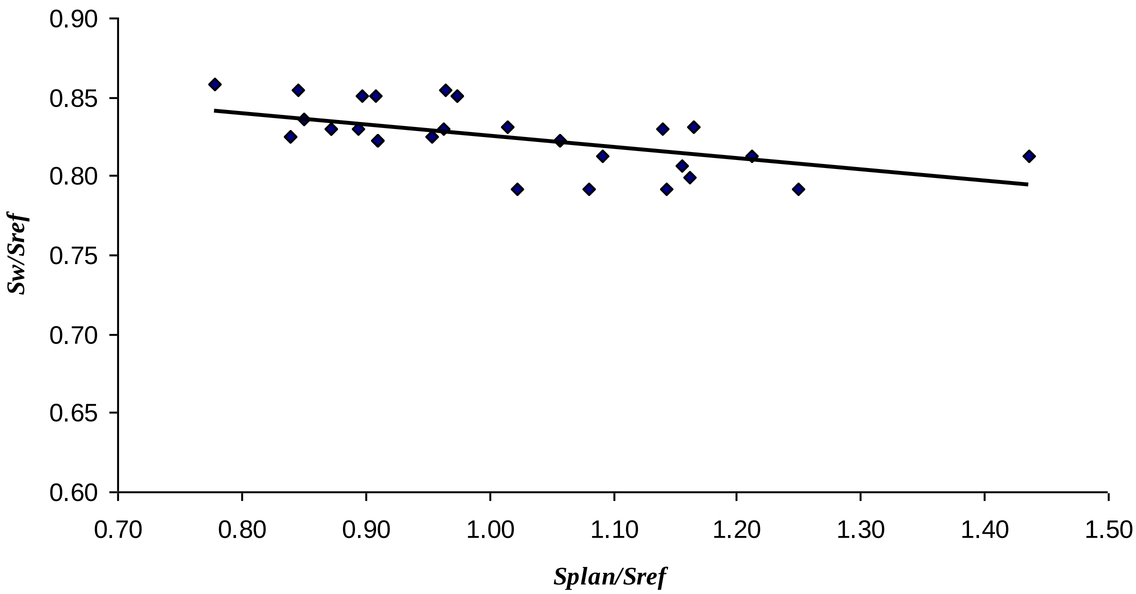


Figure 4 – The variation of the ratio of exposed wing area to gross wing area with the ratio of fuselage plan area to gross wing area.

The mass of the wing depends, primarily upon the plan area, S_{ref}, the aspect ratio, the sweep angle, the taper ratio, the thickness to chord ratio and the cruise Mach number. In general, the estimation of wing mass is a very complex process. However, since the wing mass is typically about 15% of the maximum permitted take-off mass, good results can be obtained with relatively crude correlations. For the present work, an empirical relationship of the form

$$\frac{MLS}{MMTO} \approx 0.00059\left(\frac{0.5\rho_0 a_0^2 S_{ref}}{MMTOg}\right)\left(\frac{(M_\infty + 0.05)(1+\lambda_w)}{t/c}\right)^{0.5}\left(\frac{A^{0.83}}{Cos^{1.35}\Lambda}\right)$$

is used. This is based upon the characteristics of the aircraft in the appendix and the relation includes an allowance of about 20% for the mass of the tailplane and fin. More details of the method of mass estimation is contained in references 8 and 9

2.2.3 Propulsion system

In order to overcome the resistance to motion provided by the air, an aircraft must have a propulsion system. The thrust that the engine generates must be sufficient to provide adequate take-off and climb performance, as well as providing efficient cruise performance. In order to determine the total amount of thrust required, two conditions need to be considered. These are

- The initial cruise thrust requirement. In this case, the engines are set at the maximum cruise rating, i.e. the turbine entry temperature (TET) is at its maximum continuous value, and the aircraft is at an altitude that is 2000′ higher than the cruise altitude for minimum fuel burn at the design Mach number and the Mach number is the maximum cruise valve (M_{MO}). This condition is more demanding on thrust than the design Mach number, optimum fuel burn altitude case and it provides a performance margin that is available for air traffic flexibility and safety.

And

- The take off case. Here the engines are set at their maximum take off thrust rating, i.e. the TET is at the maximum permitted for short periods of time (minutes), the aircraft must have sufficient thrust to take off from a runway of specified length and to be able to maintain the minimum specified climb gradient in the event of a single engine failure during the second stage climb phase.

In the case of a four engined aircraft, the total thrust is likely to be governed by the first condition, whilst the second condition is likely to determine the thrust for a twin engined aircraft. However, irrespective of which condition determines the size of the engines to be used, the engine "design point" corresponds the cruise conditions that give minimum fuel burn i.e. the thrust (TET) at the "design point" is less than the maximum continuous cruise thrust (TET).

The engine "design point " corresponds to a particular value of the ratio of the total temperature of the gas as it enters the turning vanes for the first turbine (TET), to the total temperature in the air intake (T_{04}/T_{02}), where

$$T_{02} = T_{\infty}\left(1+\left(\frac{\gamma-1}{\gamma}\right)M_{\infty}^{2}\right).$$

The turbine entry temperature is controlled primarily by the fuel flow rate into the combustion chamber. Therefore, for any combination of flight Mach number and atmospheric temperature (i.e. altitude), the fuel flow rate can be adjusted by altering the throttle setting to put the engine on its design point. At this condition, the engine will generate a particular thrust and, for a steady cruise, this must be equal to the drag of the aircraft at the same Mach number and altitude. This matching condition determines the size and mass of the propulsion system.

The engine overall efficiency, η_0, is defined as

$$\eta_0 = \frac{F_N V_\infty}{\dot{m}_f \, LCV},$$

where F_N is the thrust, V_∞ is the speed, $\dot{m}_f$ is the rate of fuel consumption and *LCV* is the lower calorific value of the fuel. It can be easily shown, and it is important to note, that, when the engine is operating on its design point, η_0 depends only upon the flight Mach number. In straight and level flight, the thrust developed by the engine is equal to the aerodynamic drag, *D*, experienced by the aircraft and the lift, *L*, is equal to the instantaneous mass of the aircraft, *M*, i.e.

$$\eta_0 = \frac{MgV_\infty}{\dot{m}_f \, LCV \, (L/D)}.$$

The thrust generated at the design point, F_N, varies with altitude and Mach number in the following way

$$\frac{F_N}{F_{00}} = \left(\frac{p_\infty}{p_0}\right) Function(M_\infty)$$

where F_{00} is the static thrust of the engine at sea level and the function depends upon the engine characteristics and the Mach number only.

The mass of the propulsion system - engine plus all elements associated with its installation in the aircraft- is closely related to the maximum static thrust level that can be generated at sea level. However, the maximum sea level static thrust is not usually the engine "design point" i.e. during the take-off run the value of *(T_{04}/T_{02})* is greater than the design value and the engine is allowed to "over perform" for short periods of time. For the purposes of this study, it is assumed that

$$\left(F_{00}\right)_{max} \approx 1.1F_{00} .$$

Once the maximum sea level thrust is known, it is possible to estimate the installed mass of the engine, M_{eng}, using the empirical relations given in references 2, 3 and 6 e.g.

$$\frac{M_{eng}}{MMTO} \approx \left(0.20 + 0.015BPR\right)\left(\frac{\left(F_{00}\right)_{max}}{MMTOg}\right),$$

where *BPR* is the engine by-pass ratio. Therefore the total propulsion system mass is given by

$$\frac{M_{prop}}{MMTO} \approx n\left(\frac{M_{eng}}{MMTO}\right),$$

where *n* is the number of engines.

By way of an example, for an engine in the Rolls Royce RB 211 class,

$$\eta_0 \approx 0.32M_\infty^{0.54},$$

$$\frac{F_N}{F_{00}} \approx \left(\frac{p_\infty}{p_0}\right)\left(0.84M_\infty^{0.34}\right) = \frac{q_\infty}{\left(0.5\rho_0 a_0^2\right)}\left(\frac{0.84}{M^{1.66}}\right),$$

and

$$\frac{M_{eng}}{MMTO} \approx 0.30\left(\frac{(F_{00})_{max}}{MMTOg}\right).$$

Therefore, since thrust must equal drag,

$$\frac{(F_{00})}{gMMTO} \approx 1.2M_{\infty}^{1.66}\left(\frac{0.5\rho_0 a_0^2 S_{ref}}{gMMTO}\right)Cd \approx \left(\frac{1}{1.1}\right)\left(\frac{(F_{00})_{max}}{gMMTO}\right)$$

where

$$Cd = \left(\frac{Drag}{q_{\infty}S_{ref}}\right)$$

and, with *n* engines,

$$\frac{M_{prop}}{MMTO} \approx n(0.40M^{1.66}\left(\frac{0.5\rho_0 a_0^2 S_{ref}}{gMMTO}\right)Cd$$

2.2.4 Other elements

The masses of the other components of the aircraft are either related to the maximum take-off mass or the maximum payload mass, i.e.

mass of the undercarriage ≈ 0.05 MMTO,

mass of flight controls and hydraulics ≈ 0.02 MMTO

and

mass of avionics, instruments and electronics ≈ 0.02 MMTO,

With

mass of furnishings (seats, bins, oxygen, carpets, toilets, galleys etc) ≈ 0.27 MMP,

mass of air conditioning system ≈ 0.03 MMP

mass of the auxiliary power unit ≈ 0.01 MMP

mass of operational items (passenger food, water etc) ≈ 0.13 MMP

and

mass of the crew ≈ 0.02 MMP.

Therefore, the total mass of these items is

$$M_{misc} \approx 0.09MMTO + 0.46MMP\ .$$

2.3 Mass of the Fuel

The fuel carried by the aircraft consists of four distinct elements; the taxi fuel, the mission fuel, the reserve fuel and the tankered fuel. Taxi fuel is that used to take the aircraft from the passenger loading ramp to the end of the runway prior to take off and from the end of the runway after landing to the passenger ramp at the destination. Mission fuel is the fuel that is consumed by the propulsion system from the beginning of the take off run to the end of the landing run. The sum of the taxi fuel and the mission fuel is called the block fuel. Reserve fuel is a contingency that is carried to cover unforeseen events during the flight e.g. diversion to another airfield in the event of an emergency, extra time held in the holding pattern at the destination airport, deviation from planned route to avoid weather etc.. The minimum amount of reserve fuel that must be carried carried is determined by law. Tankered fuel is the fuel, over and above the reserve fuel, that is still on board the aircraft when it lands at its destination. Tankered fuel may be carried if the cost of fuel at the destination is significantly greater that at the point of departure, i.e. the amount of tankered fuel is usually determined by operation economics.

In the case where the flight is one for which the aircraft carries the maximum permitted payload, MMP, takes off at the maximum permitted take-off mass, MMTO, and flies the maximum distance possible. There is no tankered fuel carried and the reserve fuel load is the minimum required by law. In general and to a good approximation, the minimum reserve fuel is given by

$$\frac{Mf_{res}}{MTO} \approx 0.048$$

As fuel is burned during the flight, the mass of the aircraft reduces. The rate of reduction is given by –

$$-\frac{dM}{dt} = -\frac{dM}{ds}\frac{ds}{dt} = -\frac{dM}{ds}V_{\infty} = \dot{m}_f = \frac{(MgV_{\infty})}{LCV(\eta_0 L/D)}$$

The value of $\eta_o(L/D)$ depends upon the geometry of the aircraft, the characteristics of the engine, the Mach number and the altitude at which the aircraft is flown. For a given aircraft, there is always a Mach number-altitude trajectory that gives a constant value of $\eta_o(L/D)$ in cruise. With $\eta_o(L/D)$ constant, this equation may be integrated to obtain the total fuel consumed as the aircraft cruises between two points separated by a total great circle distance, R, -

$$\frac{(MF)_{cruise}}{M_1} = 1 - EXP\left(-\frac{(g.R/LCV)}{(\eta_0.L/D)}\right),$$

where M_1 is the mass of the aircraft at the beginning of cruise.

However, any flight between two points on the surface of the Earth involves a take-off, a climb, a cruise, a descent and a landing. This means that more fuel will be consumed in flying from the departure point to the destination than would be required to cruise the same distance. If this additional fuel is designated by Δmf, where

$$\frac{\Delta mf}{MTO} = \varepsilon = 1 - k ,$$

and MTO is the mass of the aircraft at the beginning of the take-off run, the total (mission) fuel, MMF, consumed on a trip between two locations separated by a great circle distance, R, is

$$\frac{MMF}{MTO} = 1 - kEXP\left(-\frac{g.R}{LCV(\eta_o L/D)}\right)$$

In practice, ε has a value of about 0.025 and it depends upon the cruise altitude. However, it is important to note that the performance of the aircraft is very sensitive to this parameter. An approximate relationship between ε and altitude is given in reference 4. Using this as a guide, the corresponding variation of k with atmospheric pressure is

$$k \approx 1 + 0.011\left(\ln\left(\frac{p_\infty}{p_0}\right)\right)$$

Therefore, if the aircraft is taking off at the maximum permissible weight, the total fuel load is given by –

$$\frac{Mfuel}{MMTO} = 1.048 - kEXP\left(-\frac{g.R}{LCV\left(\eta_o L/D\right)}\right)$$

In all practical situations, the term within the exponential is less than -1. Therefore, without any significant loss of accuracy, the exponential function may be replaced by the first three terms of the corresponding power series expansion, i.e.

$$\frac{Mfuel}{MMTO} \approx \left(1.048 - k\right) + k\left(\frac{gR}{LCV\left(\eta_0 L/D\right)}\right) - \frac{k}{2}\left(\frac{gR}{LCV\left(\eta_0 L/D\right)}\right)^2 .$$

It is tempting to conclude from this relation that that the mission fuel, *MMF*, is lowest when $\eta_0 L/D$ has its largest possible value. However, this is not quite the case since *MMTO* is itself a function of $\eta_0 L/D$. Nevertheless, the best fuel consumption occurs when $\eta_0 L/D$ is close to its maximum value. Therefore, the assumption that the best fuel consumption occurs at maximum $\eta_0 L/D$ is useful for obtaining initial estimates.

2.4 Aircraft Drag

In aircraft studies, it is customary to use non-dimensional parameters. The most important of these are the lift coefficient, C_L, and the drag coefficient, C_d, where

$$C_l = \frac{Lift}{q_\infty S_{ref}}$$

and

$$C_D = \frac{Drag}{q_\infty S_{ref}}$$

where q_∞ is the dynamic pressure

$$q_\infty = \frac{\gamma}{2} p_\infty M_\infty^2$$

and S_{ref} is the gross wing area.

The total drag of a civil aircraft is made up of three separate components; the profile drag, the lift induced drag and the wave drag. Profile drag is the result of viscous and pressure forces acting on the parts of the aircraft that are exposed to the air through which the aircraft is flying. In this study, this component is assumed to be independent of the lift generated by the wing. Lift induced drag is produced by pressure forces that are a direct consequence of the lift force developed by the wing. Wave drag is the result of pressure forces generated when the aircraft speed approaches the speed of sound. It appears mainly on the wing and it depends upon Mach number, lift coefficient, wing sweep angle and wing thickness to chord ratio.

Therefore, the total drag force, D, is given by

$$D = q_\infty S_{ref} \left(Cd_0 + Cd_i + Cd_{wave} \right)$$

where the profile drag coefficient

$$Cd_0 = \left(Cd_0 \right)_{fus} + \left(Cd_0 \right)_{wing} + \left(Cd_0 \right)_{tail} + \left(Cd_0 \right)_{fin} + \left(Cd_0 \right)_{nacelles},$$

the lift induced drag coefficient, see references 4, is given by

$$Cd_i = \frac{1}{\pi A e} C_l^2,$$

where e is the aircraft Oswald efficiency factor ($e \approx 0.9$) and the wave drag coefficient, see reference 4, is given by

$$Cd_{wave} = Function\left(M_\infty, C_l, \left(t/c\right), \Lambda\right).$$

The equation for drag is such that, for a given aircraft geometry and flight Mach number, there is a value of lift coefficient, C_L, at which the ratio of lift to drag, C_L/Cd, and, hence, $\eta_0 L/D$ is maximum. The calculation of this condition is complicated by the presence of the wave drag. However, if the flight Mach number is less than 0.65, the wave drag is negligibly small. In this case, the optimum condition is easily determined -

$$\frac{d\left(D/L\right)}{dC_l} = \left(-1\right)\frac{Cd_0}{C_l^2} + \frac{1}{\pi Ae} = 0.$$

Hence,

$$\left(C_L\right)_{L/D\max} = \left(\pi AeCd_0\right)^{1/2},$$

$$\left(Cd\right)_{L/D\max} = 2Cd_0$$

and

$$\left(L/D\right)_{\max} = \frac{1}{2}\left(\frac{\pi Ae}{Cd_0}\right)^{1/2}.$$

When the wave drag is significant, the value of $(L/D)_{max}$ is reduced and the value of C_l at which L/D is a maximum is also reduced. However, to first order, when the wave drag is small, it can be shown that

$$\left(L/D\right)_{\max} \approx \frac{1}{2}\left(\frac{\pi Ae}{Cd_0}\right)^{1/2}\left(1 - \frac{1}{2}\frac{\left(Cd_{wave}\right)_{L/D\max}}{Cd_0}\right)$$

with

$$\left(C_L\right)_{L/D\max} \approx \left(\pi AeCd_0\right)^{1/2}\left(1 + \alpha\left(\frac{M_\infty}{\eta_0}\frac{d\eta_0}{dM_\infty}\right)\left(\pi AeCd_0\right)^{1/2} + \frac{1}{2}\frac{\left(Cd_{wave}\right)_{L/D\max}}{Cd_0}\right)$$

and

$$\left(Cd\right)_{L/D\max} \approx 2Cd_0\left(1+\alpha\left(\frac{M_\infty}{\eta_0}\frac{d\eta_0}{dM_\infty}\right)\left(\pi AeCd_0\right)^{1/2}+\frac{\left(Cd_{wave}\right)_{L/D\max}}{Cd_0}\right),$$

where

$$\alpha=\left(\frac{\left(1-0.175\right)}{Cos\Lambda\left(0.971Cos\Lambda-\left(t/C\right)\right)}\right)$$

For reasons that will be explained later in this report, an aircraft will normally be designed to operating close to, but not exactly at, the maximum L/D condition. If the aircraft is designed so that

$$\left(L/D\right)=m\left(L/D\right)_{max}$$

where the coefficient m is very close to unity, then

$$C_L \approx \frac{1}{m}\left(1-\left(1-m^2\right)^{1/2}\right)\left(C_L\right)_{L/D\,max}$$

and

$$Cd \approx \frac{1}{m^2}\left(1-\left(1-m^2\right)^{1/2}\right)\left(Cd\right)_{L/D\,max}.$$

Although the algebraic complexity has increased, it is important to recognise that the effect of compressibility effects is small. Therefore, these extra terms can be estimated using relatively crude methods without much loss of accuracy. On this basis, the value of the wave drag at the conditions for which *L/D* is maximum may be taken to be-

$$\frac{\left(Cd_{wave}\right)_{L/D\,max}}{Cd_0} \approx 0.0006\frac{Cos^3\Lambda}{Cd_0}+0.066\left(\frac{M_\infty}{\eta_0}\frac{d\eta_0}{dM_\infty}\right).$$

Therefore, for a given aircraft geometry and engine flying at a given Mach number, the condition at which *L/D* is a maximum is a function of Cd_0 only.

The zero-lift drag coefficient, Cd_0, of the complete aircraft is estimated by summing the contributions from the various components. Each sub-element is assumed to be of the form-

$$(Cd_0)_{comp} = (C_F)_{comp} \frac{(S_{wet})_{comp}}{S_{ref}} (1+\phi_{comp}),$$

where φ_{comp} is the component form factor. Hence,

$$Cd_0 = (C_F)_w \left(\left(\frac{(S_w)_{wet}}{S_{ref}} \right)(1+\phi_w) + \left(\frac{(C_F)_f}{(C_F)_w} \right) \left(\frac{(S_f)_{wet}}{S_{ref}} \right)(1+\phi_f) + \left(\frac{(C_F)_{tail}}{(C_F)_w} \right) \left(\frac{(S_{tail})_{wet}}{S_{ref}} \right)(1+\phi_{tail}) \right)$$

However, it has already been shown that the wetted areas of the fuselage and the wing are linear functions of the ratio of the fuselage plan area to the gross wing area. Therefore, the basic form of this relation is

$$Cd_0 \approx (C_F)_w \left(A + B \left(\frac{S_{plan}}{S_{ref}} \right) \right)$$

where A and B depend upon all the other geometric variables that define the shape of the aircraft. The values of $Cd_0/(C_F)_w$ have been estimated for the aircraft listed in the appendix using the methods described in references 2, 5 and 6 and the results are tabulated in Appendix 2. The variation of $Cd_0/(C_F)_w$ with S_{plan}/S_{ref} is shown in figure 5.

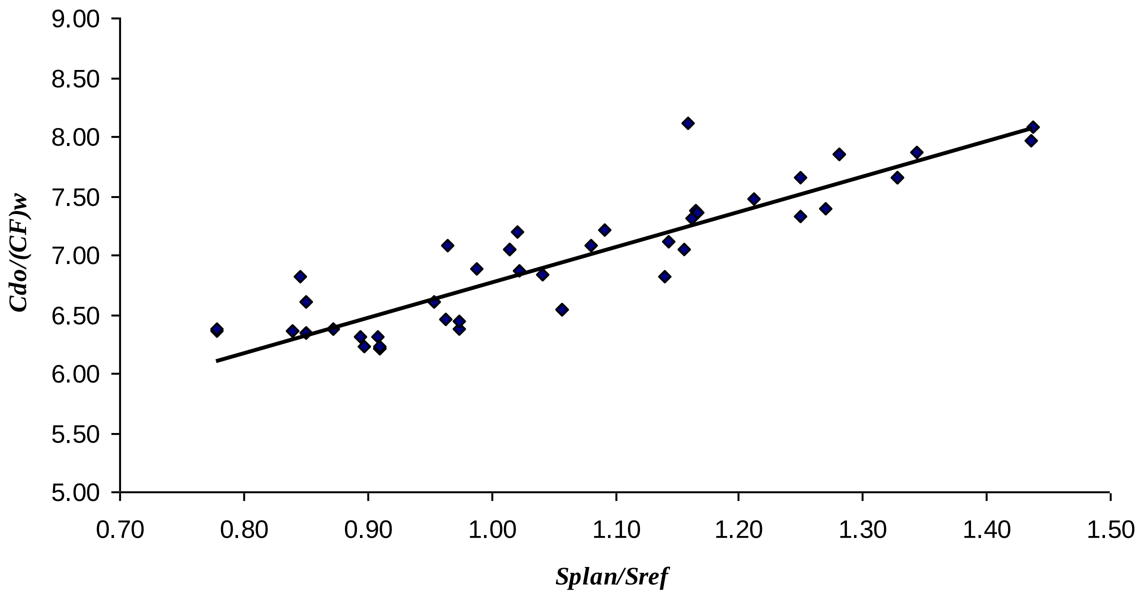


Figure 5 – The variation of zero lift drag normalised with the wing mean skin friction with the ratio of fuselage plan area to gross wing area.

A best fit to the data gives

$$Cd_0 = \left(\overline{C_F}\right)_w \left(3.7 + 3.0\left(\frac{S_{plan}}{S_{ref}}\right)\right).$$

The variation of Cd_0 with aircraft speed altitude and scale is determined by the value of the skin friction coefficient of the wing. According to reference 2, this can be taken to be

$$\left(\overline{C_F}\right)_w \approx \frac{0.455}{\left(\log_{10}\left(\frac{M_\infty a_\infty \overline{C}}{\nu_\infty}\right)\right)^{2.58}}$$

where a_∞ and ν_∞ are the speed of sound and the kinematic viscosity of air at the cruising altitude of the aircraft.

In general, the value of the mean wing chord is not known. However, the empirical data suggest that, to a first approximation, the mean wing chord is about 1.3 times the maximum width of the fuselage. This is sufficiently accurate for the estimation of the skin friction coefficient i.e.

$$(C_F) \approx \frac{0.455}{\left(log_{10}\left(\frac{1.3M_\infty a_\infty b}{\nu_\infty}\right)\right)^{2.58}}.$$

2.5 Wing area for minimum fuel burn

Consider an aircraft that is carrying the maximum possible payload, MMP, and has sufficient fuel to bring the aircraft to its maximum permitted take-off mass. If the aircraft is to fly a maximum range mission then the mass breakdown is

$$MMTO = MOE + MMP + MF_{res} + MMF$$

where MF_{res} is the mass of the reserve fuel, *MMF* is the mass of the mission fuel and *MOE* is the operational empty mass. The operation empty mass can be broken down into

$$MOE = M_{fus} + M_{LS} + M_{prop} + M_{misc}.$$

By using the relationships already developed, together with the results derived in appendix 2, it can be shown that, for an aircraft flying at a specified cruise Mach number and altitude

$$M_{fus} = a(MMP),$$

$$M_{LS} = b(S_{ref}),$$

$$M_{prop} = c + d(S_{ref}),$$

and

$$M_{misc} = e(MMTO) + f(MMP),$$

where *a, b, c,* etc are constants. Therefore,

$$MOE = (a + f)(MMP) + e(MMTO) + c + (b + d)(S_{ref}).$$

Similarly,

$$MMF = g(MMTO) + h\left(\frac{MMTO}{S_{ref}}\right)$$

and

$$MF_{res} = i(MMTO).$$

Hence,

$$MMTO = (a + f + 1)(MMP) + (e + i + g)(MMTO) + c + (b + d)(S_{ref}) + h\left(\frac{MMTO}{S_{ref}}\right)$$

and

$$\frac{MMTO}{S_{ref}}\left[S_{ref}(1 - e - i - g) - h\right] = (a + f + 1)(MMP) + c + (b + d)(S_{ref})$$

However, at the beginning of the cruise phase, the lift coefficient for minimum fuel burn is

$$C_L \approx \frac{1}{m}\left(1-\left(1-m^2\right)^{1/2}\right)\left(C_L\right)_{L/D\max} = \frac{kg}{q_\infty}\frac{MMTO}{S_{ref}}$$

and so

$$\frac{MMTO}{S_{ref}} = \frac{q_\infty}{kg}\left(j + \frac{k}{S_{ref}}\right).$$

For a given aircraft and engine combination flying at a specified altitude, these relations can only be satisfied by one value of S_{ref}, i.e. the condition that the fuel burn be minimised fixes the wing area and, hence, the complete layout of the aircraft.

2.6 Results

Figures 6,7 and 8 show sample estimates of the variation of fuel burn per unit payload per unit distance travelled with altitude for Mach numbers of 0.75, 0.8 and 0.85 and ranges of 2000, 4000, 6000 and 8000 nm for aircraft designed to carry 170, 290 and 440 passengers in a single class configuration. In terms of current aircraft, 170 passengers is typical of the Boeing 737 and the Airbus A-320, 290 passengers is typical of the Boeing 767 and the Airbus A-310, whilst 440 passengers is typical of the Boeing 777 and the Airbus A-340.

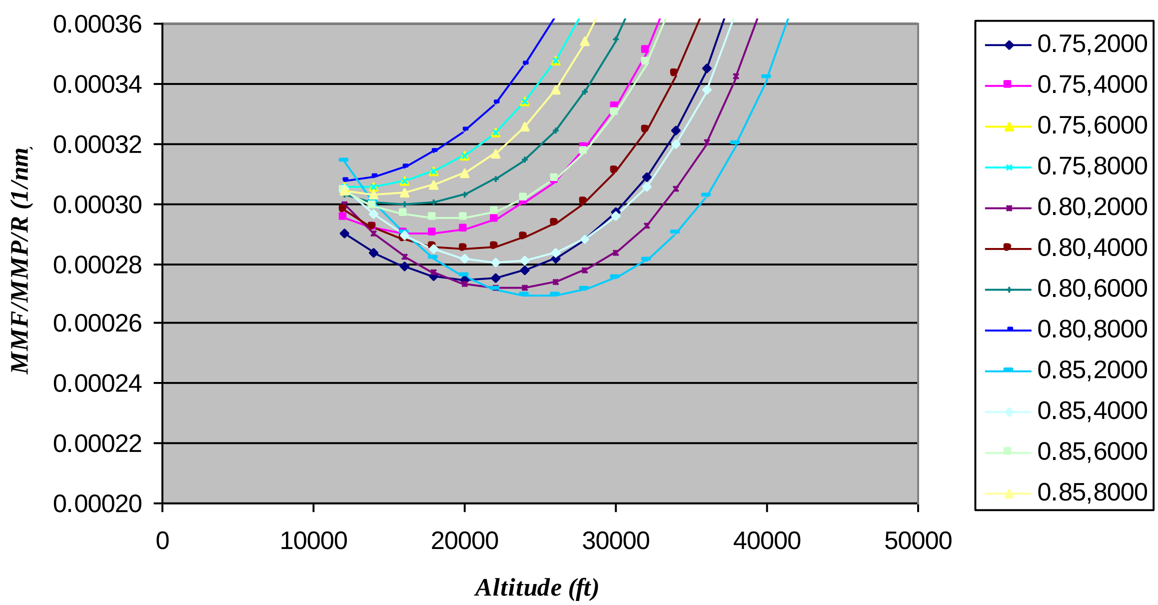


Figure 6 – Variation of fuel burn per unit payload per unit distance flown with initial cruise altitude for an aircraft with 170 single class passengers for a range of Mach numbers and ranges.

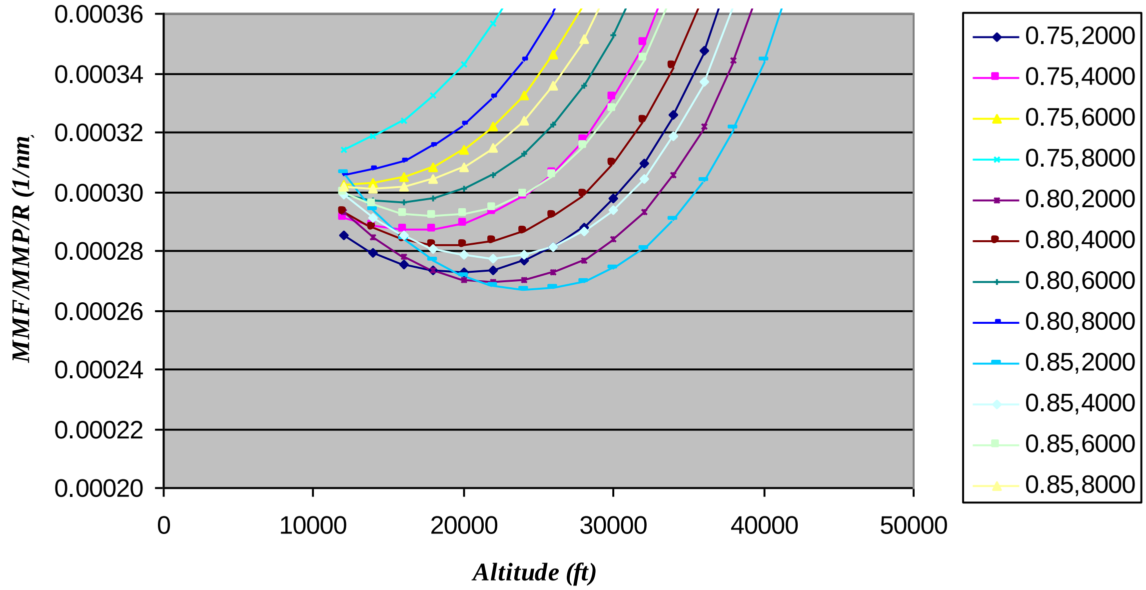


Figure 7 – Variation of fuel burn per unit payload per unit distance flown with initial cruise altitude for an aircraft with 390 single class passengers for a range of Mach numbers and ranges.

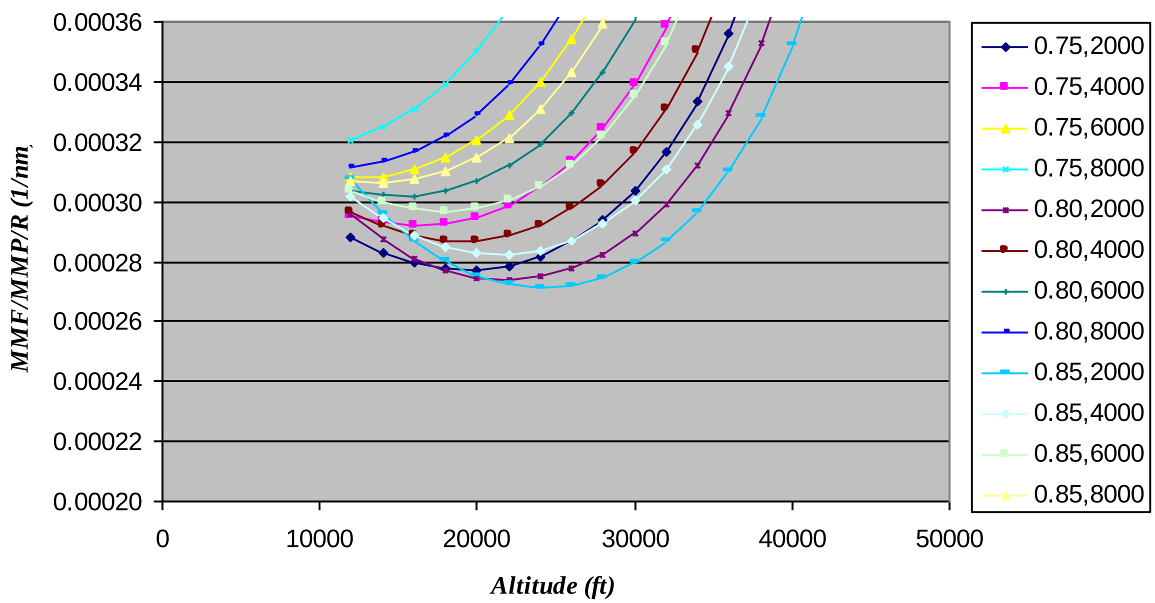


Figure 8 – Variation of fuel burn per unit payload per unit distance flown with initial cruise altitude for an aircraft with 440 single class passengers for a range of Mach numbers and ranges.

It is immediately apparent that the values of *MMF/MMP/R* are very similar for the three sizes of aircraft considered. The curves each have a minimum and the value of *MMF/MMP/R* at the minimum decreases as the Mach number increases and increases as the range increases, whilst the altitude at the minimum increases with increasing Mach number and deceases with increasing range.

However, this optimum condition is "unconstrained" and, in practise, there may be other requirements that affect the choice of the wing area. The most important of these are

- maximum fuel volume available within the wing
- threshold speed at landing
- size of the available (or design) runway

and

- the need to achieve a specified minimum climb rate following take off with one engine inoperative.

The available fuel volume is an issue for long range aircraft. However, if necessary, it is always possible to accommodate fuel in other parts of the aircraft, e.g. in the fuselage or in the tail. The take-off and landing requirements are a more serious problem.

Take-off and landing performance is subject to the airworthiness requirements of the appropriate regulatory body. For take-off, the runway must be sufficiently long for an aircraft that has suffered an engine failure during the ground run to either abandon the take-off and use the brakes to stop safely or to continue the take-off and be able to achieve a safe rate of climb. The computation of the distances required is complex. However, according to reference 4, for a twin engined aircraft that has suffered an engine failure the take off distance required to achieve a height of 35′ is approximately

$$\frac{dg}{a_0^2} \approx 1.36\left(\frac{(MMTOg)}{0.5\rho_0 a_0^2 S_{ref}}\right)\left(\frac{\rho}{\rho_0}(C_L)_{T/O\,max}\frac{(F_0)_{0.7V_{LO}}}{MMTOg}\right)^{-1}$$

where $(C_L)_{T/O\,max}$ is the maximum lift coefficient that can be achieved by the wing in the take-off configuration, i.e. with the flaps set in the take-off position, and $(F_0)_{0.7VLO}$ is the thrust developed by the engine during the ground run when the aircraft speed is 70% of that required for lift off.

From this expression, it is clear that, in order to take-off from a specific airport (d_{avail} = fixed) and recalling that the thrust of the engine during the ground run is proportional to the mass of the propulsion system, M_{prop},

$$S_{ref} \geq \frac{2.72}{d_{avil}} \frac{1}{\rho} \left(\frac{1}{(C_L)_{T/O\,max}} \right) \left(\frac{MMTOg}{(F_0)_{0.7VLO}} \right) MMTO \, ,$$

A typical maximum lift coefficient for the wing in the take-off configuration is

$$(C_L)_{T/O\,max} \approx 2.6 Cos \Lambda$$

whilst

$$\left(\frac{(F_0)_{0.7VLO}}{MMTOg} \right) \approx \frac{0.85}{0.3} \left(\frac{M_{prop}}{MMTO} \right) .$$

Hence,

$$S_{ref} \geq \frac{0.37}{d_{avil}} \left(\frac{MMTO}{\rho Cos \Lambda} \right) \left(\frac{MMTO}{M_{prop}} \right)$$

Similarly, there are airworthiness rules that apply to the landing distance required. Each aircraft has a maximum landing mass, *MLM*, specified in its type certificate and this is related to the maximum zero fuel mass, *MMZF* by

$$MLM = MOE + MMP + MF_{nc} = MMZF + MF_{NC}$$

where MF_{NC} is the fuel that has not been consumed during the flight. This extra fuel must be greater than, or equal to, the legal minimum reserve fuel required for the flight. For the aircraft listed in the appendix,

$$MF_{NC} \approx 0.085 MMTO$$

This is almost twice the minimum reserve fuel required for a maximum range trip at maximum payload and maximum take-off mass. The speed at which the aircraft lands is determined by the value of $MLM/(S_{ref}\ Cl_{max})$ and the subsequent length of the ground run depends upon MLM/S_{ref}, the characteristics of the brakes and the amount of braking applied.

An examination of the variation of $MLMg/(S_{ref}\ Cl_{max})$ with range for the aircraft listed in the appendix is shown in figure 9

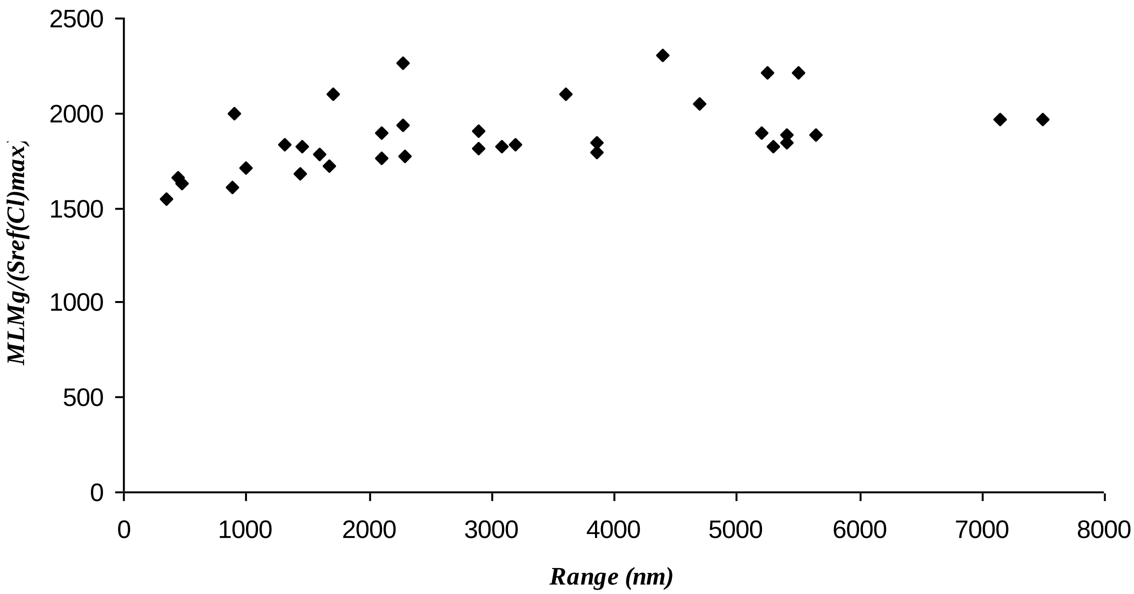


Figure 9 – The variation of $MLMg/(S_{ref}\ Cl_{max})$ with range for the aircraft listed in the appendix.

These results suggest that, to a good approximation, the design landing speed is largely independent of the design range of the aircraft. This implies that, in practise, the size of the wing is determined, or severely constrained, by the need to land a particular (safe) speed. The airworthiness requirements are that the threshold speed should be greater than 1.3 times the stalling speed with the flaps in the landing condition, i.e.

$$(q_\infty)_{approach} \geq (1.3)^2 \left(\frac{MLMg}{S_{ref}(C_L)_{max}}\right) \approx 3200\ (N/m^2).$$

Therefore, the minimum landing speed at the highest permitted landing mass is about 140 kts (70 m/sec). To meet this requirement,

$$S_{ref} \geq 0.00053\left(\frac{MLMg}{(C_L)_{max}}\right) (m^2).$$

The impact on aircraft wing area of having to meet the safe landing speed requirement is shown in figure 10. Here the loci of wing area for best fuel burn and wing area for safe landing speed are shown as functions of the cruise altitude.

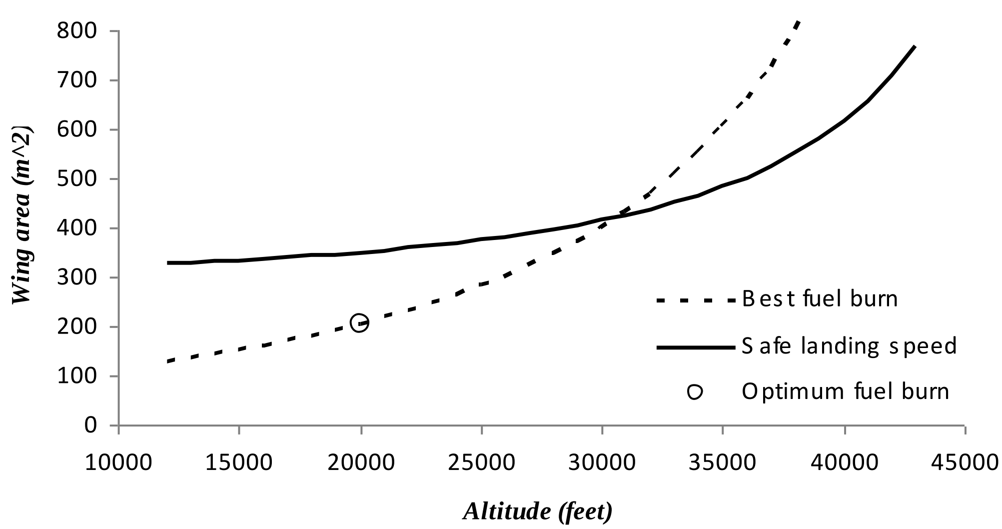


Figure 10 – The variation of *Sref* with altitude for the minimum fuel burn aircraft and for a fixed landing speed; M_∞ is 0.83 and the aircraft is designed to carry 440 passengers a distance of 3800 nm.

The results show that the landing speed requirement demands a wing whose area exceeds that of the aircraft that has the minimum fuel burn per unit payload per unit distance flown. Therefore, the practical aircraft has the wing area that corresponds to the intersection of the two lines. In the case given, this is an S_{ref} of 420 m^2 and the aircraft has a maximum take off mass of 270,000 kg. The need to fix the landing speed has caused the cruise altitude to increase by approximately 11,000 ft, moving from 20,000 ft to 31,000 ft.

The variation of fuel burn per unit payload per unit distance flown for the optimum aircraft and the aircraft with fixed S_{ref} is given in figure 11. The unconstrained aircraft has a minimum MMF/MMP/R of 0.000280 (1/nm), whilst the constrained aircraft has a minimum fuel burn of 0.000310 (1/nm), a difference of 10%.

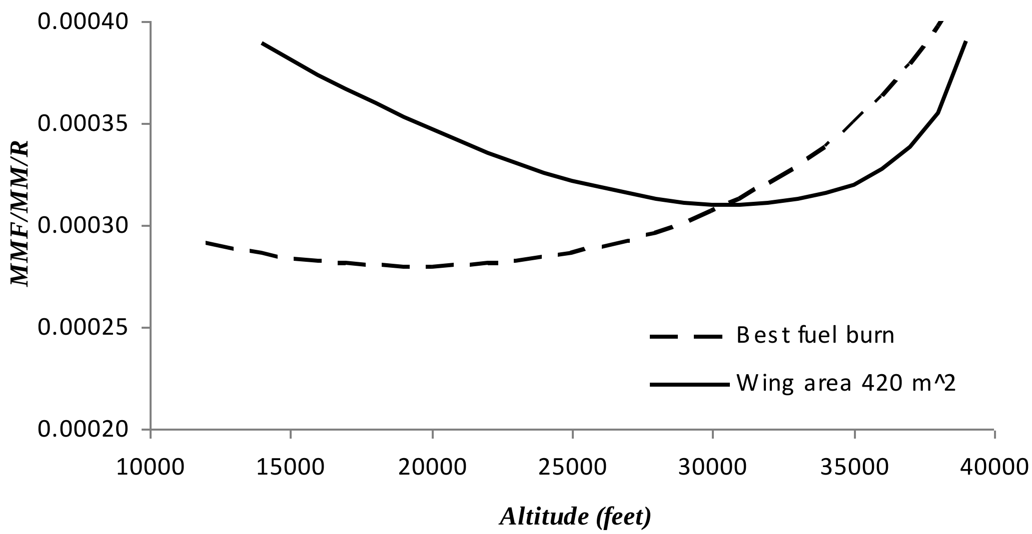


Figure 11 - The variation of fuel burn per unit payload per unit distance flown with altitude for the minimum fuel burn aircraft and for a fixed landing speed; M_∞ is 0.83 and the aircraft is designed to carry 440 passengers a distance of 3800 nm.

The solid line in figure 11 is the locus of a series of different aircraft and it is very (very close) close to the minimum fuel burn for aircraft beginning the cruise phase at the same altitude. Therefore, it is possible to quantify the fuel burn penalty incurred by requiring an aircraft to begin its cruise phase at an altitude above, or below, the optimum for the practical aircraft. This is shown in figure 12.

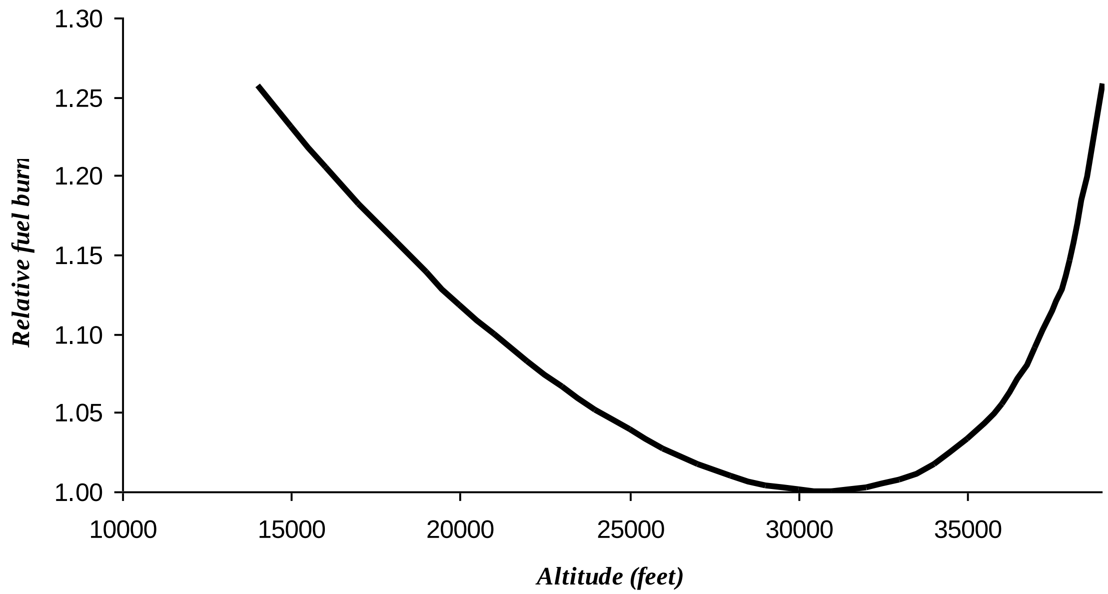


Figure 12 – The variation of the fuel burned per unit payload per unit distance flown with design altitude normalised with respect to the optimum condition; M_∞ is 0.83 and the aircraft is designed to carry 440 passengers a distance of 3800 nm.

This is a direct indication of the fuel burn, and, hence, carbon dioxide penalty, that would have to be paid if aircraft were specifically designed to cruise at lower, or higher, altitudes than the optimum in order to avoid contrail formation.

Finally, once the configuration is fixed, a specific aircraft can be operated at a range of Mach numbers and altitudes. The range of conditions that can be achieved is limited by the available engine thrust. Each possible Mach number and cruise altitude pair will correspond to a particular value of fuel burn per unit payload per unit distance flown. The corresponding variation for the "optimum" aircraft identified in this section i.e. design Mach number 0.83, design range 3800 nm, 140 kt landing speed and 440 single class passengers, is given in figure 13. The data given in the figure are based upon the assumption that the aircraft begins its journey at the maximum permitted take-off mass. In this case, the "disposable" load, which is the sum of the payload mass and the fuel mass, is constant. Hence, since the fuel required for the trip varies with the particular Mach number and altitude combination, the mass of the payload carried must also vary. This is different to the situation expressed in figure 12 where each point represents a different aircraft. In this case, each aircraft is carrying the same maximum payload. Therefore, the maximum permitted take-off mass is different in each case.

It is clear that operating the aircraft away from its design condition has major implications for the fuel burn. The penalty for changing the cruise altitude by 5,000 feet relative to the optimum is a fuel burn increase of approximately 5% and this penalty increases very rapidly if greater changes in altitude are made.

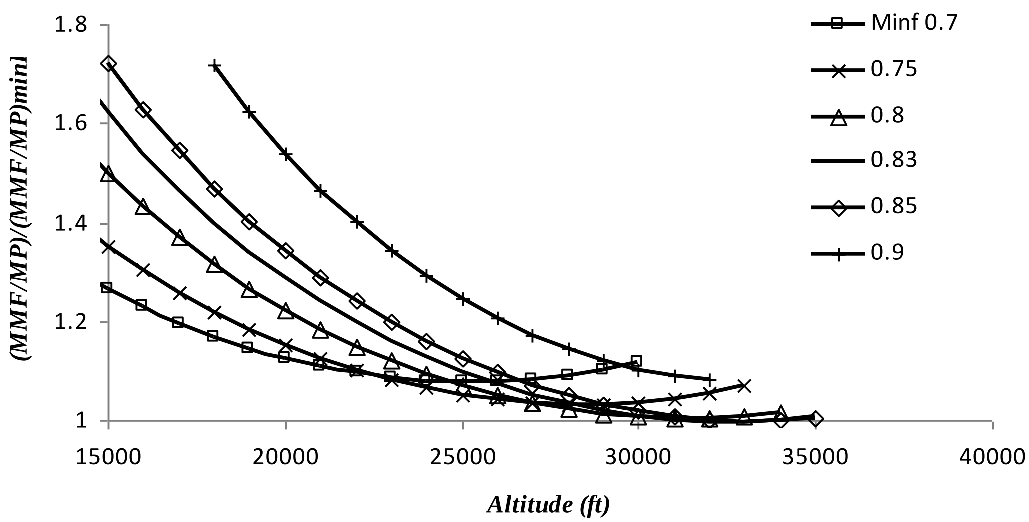


Figure 13 - The variation of fuel burned per unit payload per unit distance flown relative to the design condition with operational cruise Mach number and altitude for a fixed aircraft taking-off at the maximum permitted mass; design M_∞ is 0.83, range is 3800nm and the capacity is 440 passengers.

In the discussion so far, the cruise altitude has been the initial value, i.e. the value when the aircraft mass is largest. If the aircraft is to burn the minimum amount of fuel then both the cruise Mach number and the *L/D* must be held constant. This can only be achieved but allowing the altitude to increase steadily as the aircraft burns off fuel. Typically, the difference in height will be several thousand feet and the differential will increase as the range flown increases. Similarly, if the aircraft is operating below maximum mass the optimum cruise altitude will increase. Therefore, the range of cruise altitudes for existing aircraft lies between 30,000 and 40,000 feet.

2.7 Conclusions

It has been shown that the turbo-fan powered aircraft has an unconstrained optimum fuel burn per unit payload per unit distance flown. This optimum corresponds to a particular value of the wing area and it depends upon the design cruise Mach number, the design range and the number of passengers carried. Typically, the optimum occurs at altitudes below 25,000 feet. However, the constraints of a practical aircraft have to be satisfied. In particular, the requirement that the landing speed of the aircraft must be no greater than 140 kts places a severe constraint upon the wing area. The result is that the actual wing area must be substantially higher than the optimum fuel burn value. The implication is that the best fuel burn is increased by about 10% and the cruise altitude in increased by 10,000 feet to over 30,000 feet.

In order to minimise fuel burn, the aircraft must increase its cruise altitude as fuel is burned. The increase in cruise height is of the order 5000 feet for a cruise range of 3800 nm and increases with increasing range. Moreover, if the aircraft has a mass that is below the maximum the cruise altitudes for minimum fuel burn are increased.

All these effects combine to make cruise altitudes between 30,000 and 40,000 feet the best for fuel burn for the current aircraft fleet.

2.8 Acknowledgements

The work on aircraft modelling has benefited from the input from a number of people. BERR has provided a forum in which the ideas have been examined in detail and discussed at length. Thanks are due to Richard Pitman of BERR and to John Green and Raj Nangia and to Jeff Jupp and Les Hyde formerly of Airbus UK. In addition much help and advice has been provided by Dr. Robert Jones of Cranfield Aerospace and Dr. Howard Smith of Cranfield University and Roger Bailey, Chief Test Pilot NFLC. In addition there has been input from the members of the Greener-By-Design Executive and Technical Committees.

3 Engine

This section describes in outline the mathematical model of the turbo-fan engine. The model can be used to investigate the relationship between fuel burn rate and thrust for a range of component characteristics for both "design" and "off design" conditions. Output from the model is directly compatible with the input requirements for the aircraft model already described.

The engine has a standard gas turbine core with either a two or three shaft layout driving a fan situated in a by-pass duct. Processes through the various components of the engine are treated as one-dimensional and, since the gas passing through the core is heated to very high temperatures, "real" gas effects are taken into account. This is achieved by treating each component specie e.g. oxygen, nitrogen, water etc. separately and modelling them as a "perfect gas" i.e. the specific heat at constant pressure is assumed to be a function of temperature only. The "real" gas model is the produced by adding the individual components together to form a "mixture". This mixture is assumed to be always in thermodynamic equilibrium. In terms of model accuracy, the real gas treatment for the air and products of combustion is the step that makes an otherwise simple model very accurate indeed.

Using this approach allows any fuel to be considered; provided that its chemical composition is known and its combustion equation can be written down. In addition, it is also possible to examine the impact of water injection on the thrust of the engine and on the temperature achieved in the combustion chamber.

3.1 Core engine components

3.1.1 Intake

It is assumed that all the air passing through the engine enters via the intake. The only parameter that has to be specified is the intake isentropic efficiency.

3.1.2 Low pressure fan

It is assumed that all the air entering the engine passes through the low pressure fan. The fan design pressure ratio and the isentropic efficiency must be specified.

3.1.3 By pass duct

Downstream of the fan the engine air is divided into two streams, one passes into the engine core and the other passes in to the by pass duct. At the design condition, the ratio of the mass flow rate of air going through the by pass duct to that going into the core is called the by pass ratio. In order to estimate the flow characteristics in the by pass duct, the by pass ratio and the propelling nozzle isentropic efficiency must be specified.

3.1.4 Booster stage

Two shaft engines sometimes have a small compression stage attached to the LP shaft. Only air passing into the core part of the engine goes through the booster stage. To compute the flow characteristics, both the design pressure ratio and the isentropic efficiency must be specified.

3.1.5 Compressor stages

The design pressure ratio and the isentropic efficiency must be specified for each stage.

3.1.6 Bleed air

If air is to be bleed from the compressor section to provide power for air conditioning or de-icing, the mass flow rate and the pressure must be specified

3.1.7 Combustion chamber

An estimate must be provided for the pressure drop between the combustion chamber inlet and the outlet. The rate of fuel addition to the combustion chamber is the primary control variable for the engine. Products of combustion are determined by the equation of combustion. However, the amount of NOX produced in the engine is estimated using the empirical method described in reference 11.

3.1.8 Turbine blade cooling air

Some of the air emerging from the final stages of the compressor is fed past the combustion chamber and used to cool the nozzle guide vanes at the entry to the high pressure turbine and the turbine blades themselves. Cooling may be extended to the intermediate pressure turbine. In order to estimate the amount of cooling air needed, the maximum surface temperature must be specified. Alternatively, the actual amount of cooling air may be specified as a fraction of the total air passing through the core.

3.1.9 Turbine stages

For each turbine stage, the design pressure ratio and the isentropic efficiency must be specified. If power is being taken from the shaft for any reason e.g. the generation of electricity, this needs to be specified as well.

3.1.10 Propelling nozzle

Air from the core finally leaves the engine through a propelling nozzle. The isentropic efficiency of this component also needs to be specified.

3.2 Design point

In addition to information listed above, the altitude and the Mach number at the "Design point" must be specified. This would normally be the cruise condition for the aircraft. However, if the objective is to find the combination of airframe and engine that has the minimum fuel burn (environmental impact) for a given payload and a given range, the design point may have to be determined by an iterative process.

3.3 Output

Once the input information has been specified, the model produces the design point characteristics of a non-dimensional engine. This may be scaled to any thrust level by specifying the exit area of the core flow propelling nozzle. The key parameters are the fuel flow,

$$\frac{\dot{m}_f \left(c_p T_{01}\right)^{1/2}}{A_e p_{01}} = \alpha\left(BPR\right),$$

the gross thrust, F_G,

$$\frac{F_G + p_\infty A_e}{A_e p_{01}} = \beta\left(BPR\right),$$

and the total air flow,

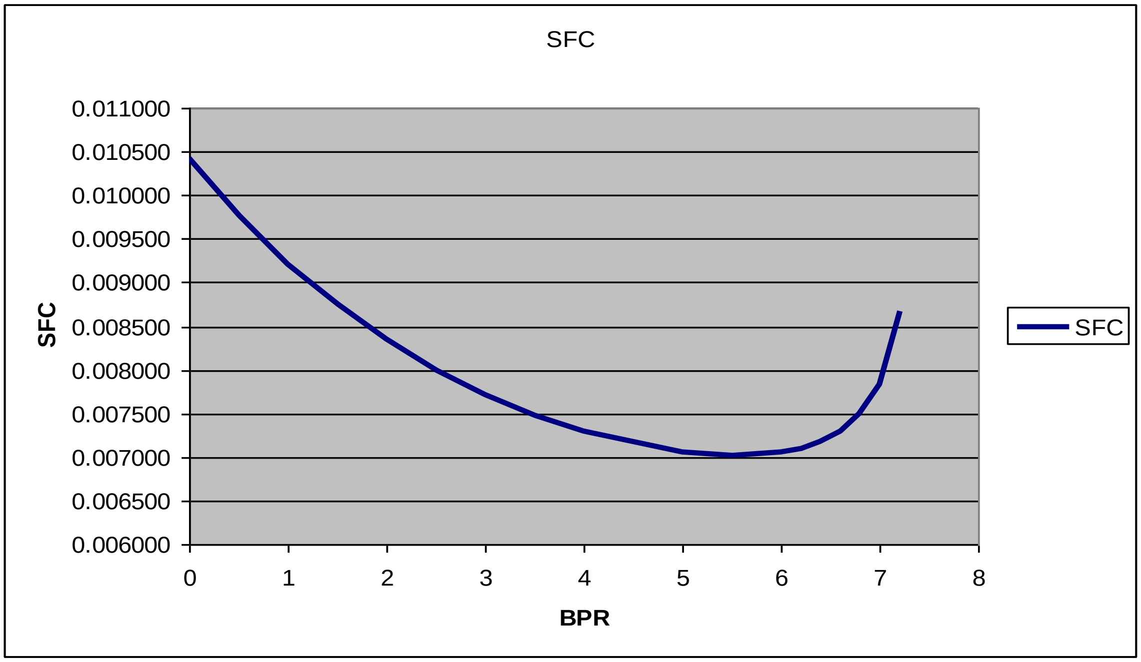
$$\frac{\dot{m} \left(c_p T_{01}\right)^{1/2}}{A_e p_{01}} = \delta\left(BPR\right).$$

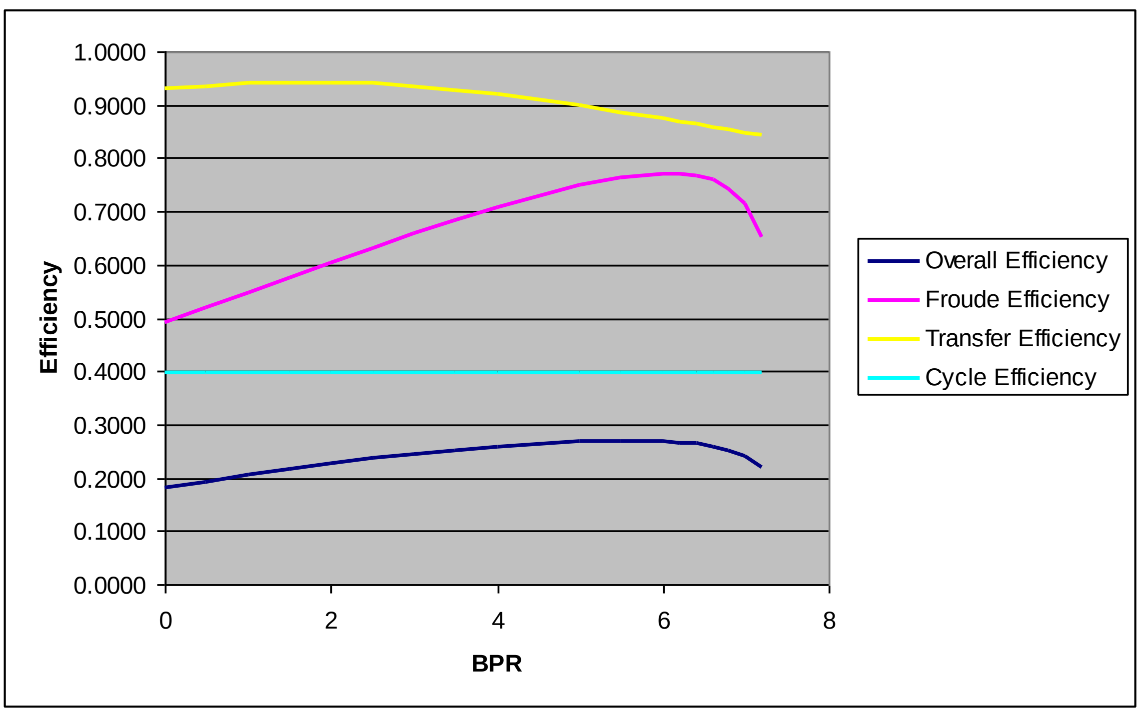
Each of the non-dimensional groups is a function of the by-pass ratio and an array of α, β and δ for a range of values of BPR is written to the aircraft model propulsion file.

The engine emission indices at the design point are also computed and written to the aircraft model propulsion file.

3.3.1 Sample output

The figures set out below show typical output from the model


SFC
SFC
0.011000
0.010500
0.010000
0.009500
0.009000
0.008500
0.008000
0.007500
0.007000
0.006500
0.006000
0 1 2 3 4 5 6 7 8
BPR
SFC


Efficiency
1.0000
0.9000
0.8000
0.7000
0.6000
0.5000
0.4000
0.3000
0.2000
0.1000
0.0000
0 2 4 6 8
BPR
Overall Efficiency
Froude Efficiency
Transfer Efficiency
Cycle Efficiency

3.4 Engine Mass

The engine mass is estimated by an empirical method developed under the Omega programme – see reference 12.

3.5 Off-Design performance

In order to estimate the off-design performance of the engine, following the guidelines set out in reference 7, it is assumed that the principle control variable is the fuel flow rate (α). In addition, the internal geometry is fixed by matching the engine components at the design point, it is assumed that the core flow is choked somewhere in the engine and that the component isentropic efficiencies are constant.

The output of this part of the model is the derivatives of the non-dimensional performance parameters β and δ with respect to α at the engine design point

$$\frac{\partial \beta}{\partial \alpha}$$

and

$$\frac{\partial \delta}{\partial \alpha}.$$

Once again, these quantities are a function of by-pass ratio and so an array of these derivatives is calculated and written to the aircraft model propulsion file.

3.6 Acknowledgements

The work on engine modelling has benefited from the input from members of Cranfield University's Department of power and Propulsion. Particular thanks are due Professor Pericles Pilidis and Dr. Panagiotis Laskaridis.

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12. Noppel, F., Emissions Model for the Omega Project

Appendix 1

Manufacturer	Type	Model	Year	MMTO (kg)
AIRBUS	A319-	100	1995	64000
AIRBUS	A320-	200	1988	73500
AIRBUS	A321-	200	1993	89000
AIRBUS	A300-	600R	1974	170500
AIRBUS	A310-	300	1983	150000
AIRBUS	A330-	200	1998	230000
AIRBUS	A330-	300	1994	212000
AIRBUS	A340-	200	1993	257000
AIRBUS	A340-	300	1994	257000
AIRBUS	A340-	500	2002	365000
AIRBUS	A340-	600	2002	365000
BOEING	737-	200	1967	52390
BOEING	737-	300	1984	56472
BOEING	737-	400	1988	62820
BOEING	737-	500	1990	52390
BOEING	737-	600	1998	56245
BOEING	737-	700	1997	60330
BOEING	737-	800	1998	70535
BOEING	747-	100	1969	333300
BOEING	747-	200	1972	371900

BOEING	747-	400	1988	362875
BOEING	747-	400 OPTION	1988	396894
BOEING	757-	200	1982	99790
BOEING	757-	200 OPTION	1982	115650
BOEING	757-	300	1999	122470
BOEING	757-	300 OPTION	1999	123605
BOEING	767-	200	1982	136078
BOEING	767-	200ER	1982	156490
BOEING	767-	200ER OPTION	1982	179170
BOEING	767-	300	1982	156492
BOEING	767-	300ER	1982	175540
BOEING	777-	200	1995	229500
BOEING	777-	200 OPTION	1995	286900
BOEING	777-	200ER		263080
BOEING	777-	200ER OPTION		297560
BOEING	777-	200LR		340194
BOEING	777-	200LR OPTION		347803
BOEING	777-	200IGW	1995	286897
BOEING	777-	300	1998	263080
BOEING	777-	300	1998	299370

		OPTION		
BOEING	777-	300ER		345047
BOEING	777-	300ER OPTION	1998	351533
DOUG.	DC8	-63	1959	158760
DOUG.	DC8	-73	1980	161025

BOEING	717-	200	1999	49895
BOEING	717-	200 OPTION		54885
BOEING	727-	200Adv	1970	95028
DOUG.		DC 9-10	1965	35245
DOUG.		DC 9-30	1967	54885
DOUG.		DC 9-40	1968	54885
DOUG.		DC 9-50	1975	54885
McDON.	/DOUG.	MD-81	1980	63503
McDON.	/DOUG.	MD-82	1980	67812
McDON.	/DOUG.	MD-83	1980	72575
McDON.	/DOUG.	MD-87	1980	63503
McDON.	/DOUG.	MD-90-30	1995	70760
DOUG.		DC10-10	1971	206384
DOUG.		DC10-30	1973	259450
McDON.	/DOUG.	MD-11	1990	283725
LOCKHEED		L1011-100	1973	211375

Appendix 2

Starting from

$$\frac{Cd_0}{\left(C_F w\right)} = \left(A + B\left(\frac{S_{plan}}{S_{ref}}\right)\right),$$

let

$$\frac{Cd_0}{\left(C_F w\right)} = \left(\frac{Cd_0}{\left(C_F w\right)}\right)_{mean} \left(1 + \tau\right),$$

where the value of $\left(\frac{Cd_0}{\left(C_F w\right)}\right)_{mean}$ is chosen so that τ is small.

It follows that

$$\tau = B\left(\frac{\left(C_F w\right)}{Cd_0}\right)_{mean}\left(\frac{S_{plan}}{S_{ref}}\right) + A\left(\frac{\left(C_F w\right)}{Cd_0}\right)_{mean} - 1$$

and, therefore.

$$\left(\frac{Cd_0}{\left(C_F w\right)}\right)^{\omega} \approx \left(\frac{Cd_0}{\left(C_F w\right)}\right)^{\omega}_{mean}\left(\left(1 - \omega + \omega A\left(\frac{\left(C_F w\right)}{Cd_0}\right)_{mean}\right) + \omega B\left(\frac{\left(C_F w\right)}{Cd_0}\right)_{mean}\left(\frac{S_{plan}}{S_{ref}}\right)\right).$$

For the aircraft list in Appendix 2, the data show that A is about 3.7, B is about 3.0 and that $\left(\frac{Cd_0}{\left(C_F w\right)}\right)_{mean}$ is about 7. In which case

$$\left(\frac{Cd_0}{\left(C_F w\right)}\right)^{1/2} \approx 2.02 + 0.57\left(\frac{S_{plan}}{S_{ref}}\right),$$

$$\left(\frac{Cd_0}{\left(C_F w\right)}\right)^{-1/2} \approx 0.47 - 0.08\left(\frac{S_{plan}}{S_{ref}}\right),$$

and

$$\left(\frac{Cd_0}{\left(C_F w\right)}\right)^{3/2} \approx 5.42 + 11.91\left(\frac{S_{plan}}{S_{ref}}\right).$$